Modeling Coupled Hydromechanical Behavior of Landfilled Waste in Bioreactor Landfills: Numerical Formulation and Validation

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Abstract: Bioreactor landfills involving leachate recirculation are emerging as the preferred option for managing municipal solid waste (MSW). Effective bioreactor landfill performance can be achieved by ensuring uniform and adequate moisture (leachate) distribution in landfilled MSW. This paper presents a numerical two-phase flow model as a tool to predict hydraulic behavior (moisture distribution and pore fluid pressures) in unsaturated MSW under leachate recirculation, mechanical response (stress-strain behavior), and coupled hydromechanical interactions of MSW in landfills. The selected mathematical model is the Fast Lagrangian Analysis of Continua (FLAC), which assumes leachate and landfill gas as two immiscible phases. The governing equations and numerical implementation are presented along with the general model implementation considerations. The model is validated by simulating the published laboratory studies, field studies, and published modeled studies. Overall, it is shown that the mathematical model is capable of providing information necessary for the design of effective bioreactor landfills by incorporating coupled hydromechanical processes. DOI: 10.1061/(ASCE)HZ.2153-5515.0000289, © 2015 American Society of Civil Engineers.

Author keywords: Bioreactor landfills; Leachate recirculation; Two-phase flow; Moisture distribution; Pore pressure; Coupled hydromechanical response.

Introduction

Landfilling is one of the most common approaches for the safe disposal of municipal solid waste. In the USA, out of 243 million tons/year of generated municipal solid waste, around 54% is disposed of in MSW landfills (USEPA 2010). Traditional MSW landfills or dry tombs have long been used in practice. However, the MSW in these landfills biodegrade at a very slow rate as a result of inadequate moisture levels, and usually results in higher costs associated with postclosure monitoring activities and overall long duration of MSW settlement (around 50 years). Recently, bioreactor landfills have been utilized as a sustainable way to dispose MSW, wherein, the leachate is recirculated into the landfilled MSW along with supplement liquids (e.g., nutrient, microbes) using different leachate recirculation systems (LRS) to enhance the overall moisture level, and therefore, increase the biological decomposition of the waste (Reinhart and Townsend 1997; Sharma and Reddy 2004).

Design of LRS, such as horizontal trenches (HTs), vertical wells (VWs), and drainage blankets (DBs) in a bioreactor landfill is an important task, as it must ensure uniform and adequate moisture distribution. The distribution of the injected leachate and consequent moisture increase depends on (1) the hydraulic properties of the MSW (including saturated and unsaturated hydraulic conductivity); (2) type of leachate recirculation system; (3) geometric formation and configuration of leachate recirculation system; and (4) spatial variation of the MSW hydraulic properties. The published literature makes simplified assumptions in assessing the leachate routing in a bioreactor landfill, including (1) saturated MSW; (2) single-phase flow; (3) homogeneous and isotropic MSW; and (4) a single set of the MSW hydraulic parameters (Kulkarni 2012; Giri and Reddy 2014b, a). Currently, the design of the leachate recirculation system is mostly based on field observations conducted at the same or similar landfills, leading to wide variations in the effectiveness of bioreactor landfills when applied in dissimilar bioreactor landfills. Some of the field studies involved monitoring with geophysical methods such as electrical resistivity tomography (ERT) imaging and neutron probes (Greiller et al. 2006; Carpenter et al. 2008). However, the data from such studies is limited and often inadequate to contribute to the successful design of leachate recirculation systems. In addition, locating LRS near side slope with relatively high injection pressure in bioreactor landfills may endanger the stability of landfill slopes due to the development of excess pore fluid pressures during the leachate injection as a result of reduced shear strength (Koerner and Soong 2000; Giri and Reddy 2014b, a).

Assessing the long-term mechanical behavior (MSW settlement/stabilization) of bioreactor landfills, initially during the waste placement as well as during leachate recirculation operations (hydromechanical interactions), is critical for designing bioreactor landfills efficiently. Landfilled MSW settlement is different from conventional soils and can range up to 40–50% of the total landfill height. It has been reported that the MSW settlement can be categorized into three phases: (1) immediate (due to applied load during waste placement); (2) primary (due to dissipation of leachate and gas from the pores of MSW and could last for few months); and
Numerical Modeling Methodologies

Since the wetting fluid wets the porous medium where the nonwetting fluid cannot, the nonwetting fluid exerts more pressure than the wetting fluid. This pressure difference causes the capillary action in the pores and creates a suction head in the pores. The capillary pressure is also a function of saturation. The computation for the flow in an unsaturated media is based on the Darcy’s flow for saturated media, which is then extended to unsaturated media by the Richards equation. The relative permeability of the liquid and gas are defined by the van Genuchten (1980) model, considering the MSW water retention characteristics.

The FLAC provides a platform to analyze the flow problem as two-phase flow in the MSW. The flow model can be analyzed as a flow-only case or it can be coupled with the mechanical calculations that compute the stress developed and the associated settlements in the system. The numerical program follows these features of the solid/fluids interaction: (1) changes in effective stress result in volumetric strain; (2) volumetric deformation causes changes in fluid pressure; and (3) Bishop effective stress is used in the detection of yield in constitutive models that involve plasticity. For application of this model, four assumptions are made: (1) MSW biodegradation and chemical reactions are complex and uncertain, and therefore, their effects are ignored and the properties of the MSW remain unchanged, since no substantial correlations are available for biodegradation and subsequent changes in the geo-technical properties of MSW; (2) landfill gas removal is not taken into account as it is assumed that bioreactor landfill has an active gas extraction system; (3) temperature effects are ignored because of lack of information regarding the effects of temperature on MSW behavior; and (4) mechanical compression is ignored in flow-only computations, as the mechanical changes associated due to fluid flow are studied comprehensively in the coupled hydromechanical study.

Governing Equations

During leachate recirculation, the unsaturated MSW pores experience simultaneous flow of the leachate and landfill gas as immiscible fluids. Based on fluid properties, the leachate is referred to as wetting fluid and landfill gas is referred to as nonwetting fluid. Since these fluids are assumed immiscible fluids, there is no mass transfer between them, as such effects may be negligible since the landfill gas is extracted immediately upon the production using active gas extraction systems at all landfills, especially bioreactor landfills. In the formulation, subscripts W and G represent leachate as wetting fluid and landfill gas as nonwetting fluid, respectively. The two-phase flow in an unsaturated MSW is influenced by degree of saturation, capillary pressure, and relative permeability or relative hydraulic conductivity for the two immiscible fluids. The saturation is defined as the fraction of the void volume of the MSW being filled by that phase. The transport of leachate (with superscript w) and gas (with superscript g) fluid is described by Darcy’s law

\[ q^w_i = -k^w_{ij} \frac{\partial P_w}{\partial x_j} (P_w - \rho_w g_i x_k) \]  
\[ q^g_i = -k^g_{ij} \frac{\mu_g}{\rho_g} \kappa^g_j \frac{\partial P_g}{\partial x_j} (P_g - \rho_g g_i x_k) \]

where \( k_{ij} \) = saturated mobility coefficient (tensor) defined as the ratio of intrinsic permeability to dynamic viscosity; \( i \) = number of zones in horizontal (x) direction; \( j \) = number of zones in vertical (y)
direction; \( \kappa_r \) = relative permeability for the fluid (function of saturation); \( \mu \) = dynamic viscosity; \( P \) = pore pressure; \( \rho \) = fluid density; and \( g \) = gravity.

Relative permeabilities are related to saturation \( (S_w) \) and are expressed by van Genuchten functions

\[
\kappa_r = S_r^\theta \left[ 1 - \left( 1 - S_r^{1/\alpha} \right)^{2} \right] \\
(3)
\]

\[
\kappa_r = 1 - S_r \left[ 1 - S_r^{1/\alpha} \right]^{2 \alpha} \\
(4)
\]

\[
S_e = \frac{S_w - S_r}{1 - S_r} \\
(5)
\]

where \( \alpha \), \( b \), and \( c \) = constant parameters for van Genuchten function; \( S_e \) = effective saturation; and \( S_r \) = residual leachate saturation.

Capillary pressure is related to the pressure difference between the leachate and gas as

\[
P_g - P_w = P_c(S_w) \\
(6)
\]

where \( P_g \) = pressure created by gas fluid; \( P_w \) = pressure created by leachate; \( P_c(S_w) \) = capillary pressure, which is a function of degree of saturation \( (S_w) \).

The sum of the saturation of leachate \( (S_w) \) and gas \( (S_g) \) should be

\[
S_w + S_g = 1 \\
(7)
\]

Fluid balance laws for the slightly compressible fluids give the variation of fluid content (variation of fluid volume per unit volume of porous material) with respect to the volumetric fluid source intensity. They are given by

\[
\frac{\partial \xi_w}{\partial t} = - \frac{\partial q_{w}^W}{\partial x_i} + q_{v}^W \\
(8a)
\]

\[
\frac{\partial \xi_G}{\partial t} = - \frac{\partial q_{G}^G}{\partial x_i} + q_{v}^G \\
(8b)
\]

where \( \xi \) = variation of fluid volume per unit volume of porous material; and \( q_v \) = volumetric fluid source intensity.

Constitutive laws for fluids are solved for the pressures in leachate and gas fluids and saturation in leachate and gas

\[
S_w \frac{\partial P_w}{\partial t} = \frac{K_w}{n} \left[ \frac{\partial \xi_w}{\partial t} - n \frac{\partial S_w}{\partial t} - S_w \frac{\partial \varepsilon}{\partial t} \right] \\
(9a)
\]

\[
S_g \frac{\partial P_G}{\partial t} = \frac{K_G}{n} \left[ \frac{\partial \xi_G}{\partial t} - n \frac{\partial S_G}{\partial t} - S_G \frac{\partial \varepsilon}{\partial t} \right] \\
(9b)
\]

By combining these equations with fluid balance laws

\[
n \left[ \frac{S_w \frac{\partial P_w}{\partial t} + \partial S_w}{K_w \frac{\partial t}{\partial t}} \right] = - \left[ \frac{\partial q_{w}^W}{\partial x_i} + S_w \frac{\partial \varepsilon}{\partial t} \right] \\
(10a)
\]

\[
n \left[ \frac{S_g \frac{\partial P_G}{\partial t} + \partial S_G}{K_G \frac{\partial t}{\partial t}} \right] = - \left[ \frac{\partial q_{G}^G}{\partial x_i} + S_G \frac{\partial \varepsilon}{\partial t} \right] \\
(10b)
\]

This gives a nonlinear system of four equations in terms of four unknowns \( P_w, P_G, S_w, \) and \( S_G \). In the fluid flow only calculation, the volumetric deformation (mechanical) term \( \partial \varepsilon / \partial t \) is ignored.

The balance momentum for two-phase flow calculations is

\[
\rho = \rho_d + n(S_w \rho_w + S_g \rho_G) \\
(11)
\]

where \( \rho_w \) and \( \rho_G \) = fluid densities; and \( \rho_d \) = matrix dry density.

For the two-phase flow problem, mechanical constitutive laws give the change in effective stress for the porous media as

\[
\Delta \sigma_{ij} = \Delta \sigma_{ij} + \Delta \bar{P} b_{ij} \\
(12)
\]

where

\[
\Delta \bar{P} = S_w \Delta P_w + S_g \Delta P_G \\
(13)
\]

From these relationships, provided total stress remains constant, deformation will occur only if a change in pore pressure takes place. For the constitutive models involving plasticity, the Bishop effective stress is used to detect the failure. The Bishop effective stress is defined as

\[
\sigma_{ij} = \sigma_{ij} + \tilde{P} + \tilde{b}_{ij} \\
(14)
\]

where

\[
\tilde{P} = S_w P_w + S_g P_G \\
(15)
\]

If needed, the strain rate and velocity gradient are related by

\[
\xi_{ij} = \frac{1}{2} \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right] \\
(16)
\]

**Numerical Implementation**

The governing fluid flow equations are solved in FLAC by discretization and the finite difference method. In a two-phase flow problem, the computation requires the two pore pressures \( P_w \) and \( P_G \) and one saturation value \( S_w \) to be specified at each node. These values vary linearly in the other nodes of a triangular element. The nodal flow values are computed on zone basis using the upstream weighing technique. The resulting differential equations are solved based on the explicit time function.

**Nodal Formulation of Fluid Balance Laws**

For a finite volume, the balance laws for leachate and gas fluid are given by

\[
S_w \frac{\Delta P_w}{K_w} + \Delta S_w = - \frac{1}{nV} (Q_w \Delta t + S_w \Delta V) \\
(17a)
\]

\[
S_g \frac{\Delta P_G}{K_G} + \Delta S_G = - \frac{1}{nV} (Q_G \Delta t + S_G \Delta V) \\
(17b)
\]

where \( V \) = nodal volume; and \( Q = \) nodal flow rate.

By capillary pressure law and saturation law, the increment in the pressure and saturation can be expressed as

\[
\Delta P_G - \Delta P_w = P'_c \Delta S_w \\
(18)
\]

\[
\Delta S_w + \Delta S_G = 0 \\
(19)
\]

where \( P'_c \) = derivative of \( P_c(S_w) \) with respect to \( S_w \)
Finally, the equations are derived for the $\Delta P_w$ and $\Delta S_w$ as
\[
\Delta P_w = -\frac{\Delta t}{nVD} \left[ Q_w \left(1 - \frac{S_G P_c^G}{K_G}\right) + Q_G \right] - \beta \frac{\Delta V}{nVD} \left[ S_w \left(1 - \frac{S_G P_c^G}{K_G}\right) + S_G \right]
\]
\[
\Delta S_w = -\frac{\Delta t}{nVD} \left[ Q_w \frac{S_G}{K_G} - \frac{Q_G}{K_G} \right] - \beta \frac{\Delta V}{nVD} S_G S_w \left[ \frac{1}{K_G} - \frac{1}{K_w} \right]
\]
where
\[
S_G P_c^G = -\frac{\rho_c g}{\alpha} \left( \frac{1-a}{a} \right) \left( 1 - S_e \right) S_e^{1/a} \left( 1 - S_e^{1/a} \right)^{-a}
\]
\[
D = \frac{S_w}{K_w} + \frac{S_G}{K_G} - \frac{S_G}{K_G} \frac{S_G P_c^G}{K_G}
\]
with $\beta$ being an undrained coefficient and equal to one (1) for the coupled hydro-mechanical calculation and zero (0) for the flow only computations. The expression for $S_G P_c^G$ becomes zero when the leachate saturation reaches 100%. The new leachate pressure and saturation time increment of $t + \Delta t$ are evaluated from those at time $t$ using the increment leachate pressure and saturation Eqs. (20) and (21), respectively, as given above, and the new value of $P_G$ at the respective node is computed based on the $P_w$ and $S_w$ values using the capillary pressure laws.

**Nodal Flow Rate**

The zone contributions to nodal flow rates $Q_w$ and $Q_G$ are calculated in FLAC using the element leachate-fluid stiffness matrix $[M_w]$. The components of this matrix are the same as those that would have been calculated for the saturated flow of the leachate fluid. For the two-phase flow, the nodal volumetric flow rates in a zone $Q_w$ and $Q_G$ are related to nodal pore pressures $P_w$ and $P_G$ by the matrix expressions
\[
Q_w = \tilde{k}_w^V [M_w] (P_w - \rho_w x_i g_i)
\]
\[
Q_G = \tilde{k}_G^V [M_w] (P_G - \rho_G x_i g_i)
\]
where $\tilde{k}_w$ and $\tilde{k}_G$ are the permeability of the zone. The upstream weighing technique is used in the numerical modeling wherein, the average saturation at inflow nodes is used with the combination of equations for relative permeabilities for the fluids.

**Time Step**

Time step represents the actual time taken for the model to conduct the flow calculations and greatly affects the number of simulations carried out per unit time. In a two-phase flow calculation, the time step for numerical stability is calculated on the basis of fluid diffusivity, even in coupled simulations. It is taken as the minimum value of stable time steps that would be used for saturated flow of the leachate and gas fluid, respectively. Time step magnitude can be estimated by (FLAC)
\[
\Delta t = L_z^2 n_{\text{min}} \frac{1}{k_w K_w} \frac{1}{k_G K_G}
\]
where $L_z$ is the smallest zone in the simulation.

The effect of increased mechanical stiffness is incorporated in quasistatic analysis in the density-scaling scheme in FLAC; the apparent mechanical bulk modulus of a zone is modified by the presence of fluids with respect to the porosity $n$ of the porous MSW by
\[
K = K + \left( S_w K_w + S_G K_G \right) \frac{S}{n}
\]

**Mechanical Coupling**

In the FLAC formulation, any of the in-built constitutive models can be used in conjunction with a numerical two-phase fluid flow calculation to model coupled hydromechanical behavior. A constitutive model calculates an effective stress change using a mechanical constitutive law, which then allows for the corresponding total stress change to be computed. Further, the mean zone pressure is evaluated in this formulation by using mean zone values for saturations and pressure increments. In transient fluid-mechanical calculations, volumetric deformations generate pore pressures and saturation changes. This hydromechanical coupling effect is taken into consideration by incorporation of the second terms in Eqs. (20) and (21) in the numerical scheme by accounting for the value of the undrained coefficient ($\beta$) as one. The nodal contributions of those terms are calculated after the distribution of zone volumetric strains to the nodes.

**Modeling Hydraulic Response**

**Moisture Distribution Using Leachate Recirculation Systems**

The capability of the numerical two-phase model is assessed by its ability to accurately predict leachate distribution, generation and distribution of pore fluid pressures in a simplified bioreactor landfill. The landfill and leachate recirculation systems are modeled using a grid in order to apply finite difference formulation and predict moisture distribution. Sensitivity analyses of different grid sizes are performed to determine the optimal grid size that allows the maximum leachate distribution within a reasonable computation time period.

Horizontal trenches, vertical wells, and drainage blankets are commonly used for leachate recirculation in a bioreactor landfill. Currently, the design and operation of these systems is based on empirical and observational approaches. The MSW is a heterogeneous and anisotropic medium that complicates the goal of achieving uniform distribution of leachate. Therefore, it is imperative to determine the moisture distribution knowing that the MSW is a heterogeneous and anisotropic unsaturated medium. The ultimate goal is to develop guidance to rationally design effective leachate recirculation systems by taking into account the coupled hydromechanical behavior of MSW.

**Initial and Boundary Conditions**

Among the conditions and values that should be specified are the initial and boundary hydraulic and mechanical conditions and the initial values for leachate fluid saturation $S_w$ and pore water pressure $P_w$. The initial value of the landfill gas pore pressure is derived from the capillary pressure law and the pore pressure or the flux boundary conditions can also be specified for each fluid. If nothing is specified for either fluid, the boundary is assumed to be impermeable for that fluid. To impose seepage condition, the nonwetting landfill gas pore pressure is ambient (value zero), and the boundary remains impermeable to the leachate as long as the pore water pressure $P_w$ is less than the ambient value (atmospheric pressure). To simulate the leachate collection and removal system, located at the
bottom of the landfill above the liner, the bottom-most grid points are fixed to zero pore pressure and the outflow rate is computed. Since this section only considers flow computations, the mechanical computations are not performed, and are shown in the subsequent sections. Lateral deformation is fixed to zero and the deformation in the base of the cell is fixed to zero, whether lateral or vertical.

**Model Parameters**

The model considers the MSW in its different forms as homogeneous or heterogeneous and isotropic or anisotropic material. For any of these conditions, the common initial model parameters that are needed with their typical values are listed in Table 1. The initial pore water pressures are assumed to be zero at all grid points, as no significant leachate is present in the MSW. The leachate and landfill gas bulk modulus values used are the typical values for water and air, respectively. The unsaturated hydraulic parameters of the MSW are represented by the van Genuchten fitting model (1980) and include the saturated hydraulic conductivity, residual saturation, saturated moisture content, matric suction, and fitting parameters \(a\), \(b\), \(c\), and \(P_0\). The coefficient for the pore water pressure increment due to the volumetric strain is assumed to be zero to represent flow-only calculations, and volumetric strains due to mechanical effects are not computed in this section. The initial porosity value of the MSW is also needed; it is dependent on the biodegradation and unit weight of the MSW. The viscosity ratio is defined as the ratio between the dynamic viscosity of leachate (wetting fluid) and landfill gas (nonwetting fluid), and these viscosities are assumed to be the same as those for water and air, respectively. In the absence of enough data, the physical properties of leachate are assumed to be the same as water; similarly, the biogas (nonwetting fluid) was considered to have same physical properties as air. Similar assumptions have successfully been reported elsewhere (Haydar and Khire 2005).

The hydraulic properties of the MSW can significantly influence the model results. Several studies reported on the saturated hydraulic conductivity of the MSW, but only few are conducted on the unsaturated hydraulic conductivity of the MSW. These hydraulic properties vary significantly with composition, density (overburden pressure), and rate of biodegradation. The saturated hydraulic conductivity of the MSW is reported to vary from \(10^{-2}\) to \(10^{-7}\) cm/s (Reddy et al. 2009a). The saturated hydraulic conductivity values selected for the evaluation of the grid size sensitivity vary from \(10^{-3}\) to \(10^{-5}\) cm/s. Based on the reported laboratory and field test results, the saturated hydraulic conductivity is related to normal function by the following equation (Reddy et al. 2009a):

\[
k_e = k_{eo} \left[ 1 + \left( \frac{\sigma'}{P_a} \right) \right]^{-b}
\]

where \(k_{eo}\) = initial saturated hydraulic conductivity at zero normal pressure in MSW; \(\sigma'\) = effective normal pressure; \(P_a\) = atmospheric pressure; and \(B\) = heterogeneity constant, typical value of 5.3. The unsaturated hydraulic parameters of the MSW based on limited published studies are summarized in Table 2.

**Grid Size Analysis**

**Model Application**

The model’s grid size can influence the modeling outcomes. In order to assess this, a bioreactor landfill cell of \(10 \times 10\) m with horizontal trench as leachate recirculation system is considered...
Table 2. Published Unsaturated Hydraulic Properties of the MSW according to Stoltz et al. (2012)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>MSW A-1</th>
<th>MSW A-2</th>
<th>MSW A-3</th>
<th>MSW A-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit weight (kN/m³)</td>
<td>4.5</td>
<td>5.3</td>
<td>6.08</td>
<td>7.55</td>
</tr>
<tr>
<td>Porosity</td>
<td>0.692</td>
<td>0.62</td>
<td>0.586</td>
<td>0.453</td>
</tr>
<tr>
<td>Matric suction α (kPa)</td>
<td>3.0</td>
<td>2.9</td>
<td>2.3</td>
<td>0.57</td>
</tr>
<tr>
<td>Residual moisture content θᵣ</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>Saturated moisture content θₛ</td>
<td>0.692</td>
<td>0.62</td>
<td>0.586</td>
<td>0.453</td>
</tr>
<tr>
<td>van Genuchten ‘a’</td>
<td>1.63</td>
<td>1.56</td>
<td>1.47</td>
<td>1.33</td>
</tr>
<tr>
<td>van Genuchten ‘b’</td>
<td>0.39</td>
<td>0.36</td>
<td>0.32</td>
<td>0.25</td>
</tr>
<tr>
<td>van Genuchten ‘c’</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Results

Fig. 3 illustrates the typical saturation contours in the bioreactor landfill cell for different square grid sizes for the leachate injection flow rate of 4 m³/d/m and saturated hydraulic conductivity of the MSW of 10⁻⁵ cm/s. Fig. 4 shows the wetted area of the different grids under the different injection flow rates and hydraulic conductivity of the MSW. The saturation contours indicate that the larger or coarser grid, which measures as 1 m, produces the greatest wetted area under all three injection flow rates. The saturation contours and wetted area under different leachate injection rates indicate that the grid size plays an important role to compute the change in saturation in each grid. The computed change in saturation in each grid is the value with respect to each grid size and is not related to the whole model size (FLAC). The percentage change in the wetted area is computed for each grid size and the values are compared with the finest mesh when the model inputs are varied for saturated hydraulic conductivity and leachate inflow. The effect of the injection flow rate shows a considerable variation (including in excess of 100%) in the wetted area when the grid size is 1 or 0.5 m, and the difference reduces drastically when the square grid size is 0.3 m (dropping from 200% for the grid of 1 m to 13% for grid of 0.3 m) is adopted [Fig. 4(a)]. Further, the results for the effect of saturated hydraulic conductivity show similar results on the wetted area [Fig. 4(b)]; i.e., when the grid size is 1 m, the percentage error is significant (>200%) for mesh size of 1 and 0.5 m, and the error drops to less than 25, 18, and 8% for kₛₚ of 10⁻³, 10⁻⁴, and 10⁻⁵ cm/s at the grid size is 0.3 m. This reduces further for the finer meshes. As the grid size decreases, the wetted area decreases and for the square grids less than 0.3 m, the overall percentage change in the wetted area is ranges between 15 and 5%, when compared with the finest mesh.

The pore water pressure distribution in the landfill cell is plotted in Figs. 5 and 6 for the different grid sizes, leachate injection flow rate, and saturated hydraulic conductivity of the MSW. The larger grid size (1 m) resulted in the high maximum pore water pressure development (around 225 kPa for kₛₚ = 10⁻⁵ cm/s), as shown in Fig. 5. The maximum pore water pressure distribution in these figures indicates that the area of influence of the maximum pore water pressure developed in the system is dependent on the rate of leachate injection. The higher the injection rate, the greater the pore water pressure will be [Figs. 6(a and b)]. This trend repeats for all of the grids, regardless of size. However, the maximum pore

Model Parameters

Homogeneous and isotropic MSW was considered in a single layer compacted to dry unit weight of 6.08 kN/m³ with an initial 44% volumetric moisture content. The HT was located at 7 m above the drainage layer in the middle of the cell (Fig. 1). Modeling was done with the saturated hydraulic conductivity of the MSW of 10⁻³, 10⁻⁴, and 10⁻⁵ cm/s. The leachate and landfill gas fluid bulk modulus assumed are the typical values for water and air to represent leachate and landfill gas, respectively. The van Genuchten fitting parameters selected include the residual saturation, saturated moisture content, matric suction, fitting parameters a, b, c and Pₒ, found to represent the typical values for MSW at a French bioreactor landfill (Stoltz et al. 2012). The coefficient for the pore water pressure increment due to the volumetric strain is assumed as zero to represent flow only calculations. The initial porosity of the MSW was computed based on the typical moisture content and unit weight (Stoltz et al. 2012). The viscosity ratio is defined as the ratio between the viscosity of leachate fluid and gas fluid. The viscosities of leachate and landfill gas assumed are the typical values for water and air. Typical values of the elastic (bulk modulus and shear modulus) and plastic parameters (cohesion, tension, friction angle, and dilatation angle) that are selected are summarized in Table 1. The plastic properties for the Mohr-Coulomb model are selected from the published literature (Reddy et al. 2009b). All the initial model parameters for the two-phase flow analysis are summarized in Table 1, and the additional initial model parameters for the grid size analysis appear in Table 1.
Square Grid Size = 1.0 m

Square Grid Size = 0.5 m

Square Grid Size = 0.3 m

Square Grid Size = 0.25 m

Square Grid Size = 0.2 m

Square Grid Size = 0.1 m

10 m

10 m

Fig. 2. Model discretization with different grid sizes

Grid Size = 1.0 m

Grid Size = 0.5 m

Grid Size = 0.3 m

Grid Size = 0.25 m

Grid Size = 0.2 m

Grid Size = 0.1 m

10 m

10 m

Fig. 3. Saturation contours (1 week flow) \( k_{\text{sat}} = 10^{-5} \text{ cm/s} \); \( Q_i = 4 \text{ m}^3/\text{d/m} \) for 1 week continuous flow (assumed homogeneous and isotropic MSW)
water pressure developed remains more or less the same when the grid size drops below 0.3 m, for all injection flow rates. The analysis of the pore water pressure distribution for the effect of saturated hydraulic conductivity illustrated that the lower the $k_{\text{sat}}$, the higher the wetted area and pore water pressure. Interestingly, the pore water pressure development in the cell for higher $k_{\text{sat}} = 10^{-3} \text{ cm/s}$ did not show any positive pore water pressure since the injected leachate migrated so quickly toward the leachate collection system at bottom and resulted in the development of negative pore water pressure in the system [Fig. 6(b)]. Conversely, when the saturated hydraulic conductivity is reduced to a magnitude of order of 10, a sudden built up of pore water pressure is seen because of the lesser volume of pores present in the system. Comparing these values with respect to the finest mesh indicates that the pore water pressure did not vary considerably when the square grid size was 0.3 m or smaller.

Outflow rate collected at the leachate collection system, at the bottom of the landfill cell, is computed for the different grid sizes and leachate injection flow rates (Fig. 7). When the leachate injection flow rate is varied for different grid sizes, the results of the 1-week flow denote that the steady-state condition is generated rapidly when the grid size is 1 or 0.5 m. In all of the cases, the steady-state condition (where inflow rate equals outflow rate) is achieved. However, the time for the steady-state condition to be achieved is less than 1 day when the grids are coarser or larger, whereas the steady state was achieved within 4–5 days for different leachate injection rates for the finer grid size of 0.3 m or less.

The results for the computational time required for 1 week flow for the different grid sizes are also compared (Fig. 8). The computational time is based on the time step that the model assumes and is dependent on the number of grids, size of the smallest zone, saturated mobility coefficient, and fluid bulk modulus for leachate

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**Fig. 4.** Wetted area comparison with respect to: (a) effect of leachate injection rate; (b) saturated hydraulic conductivity of MSW (assumed homogeneous and isotropic MSW) for 1 week continuous flow

**Fig. 5.** Pore water pressure distribution (1 week flow) $k_{\text{sat}} = 10^{-5} \text{ cm/s}$; $Q_i = 4 \text{ m}^3/\text{d/m}$ for 1 week continuous flow (assumed homogeneous and isotropic MSW)
and landfill gas (FLAC). The computational time increases with as the mesh becomes finer (or decreases in grid size). The computation time under different injection flow rates did not vary. The reason for this is that the time step that is computed is not a function of the leachate injection flow rate. However, the time required for calculations using the finer mesh increased significantly as compared with the coarser mesh (203 min for a grid size of 0.1 and 1.56 min for a grid of 1 m). Varying the saturated hydraulic conductivity resulted in great variations in the computation time required. The finest mesh, calculated to have 100,000 open in the grid, require more than 8 hour of computational time for 1 week flow. Therefore, selection of the grid size of 0.3 m is considered to represent a reasonably short computation time for this study.

Overall, wetted area values compared with respect to the finest mesh for different grid sizes showed results ranging between 17 and 12% for different injection flow rates and 25–7% for different saturated hydraulic conductivity of the MSW. Changes in the maximum pore water pressure for different injection rates and saturated hydraulic conductivity when compared with the finer mesh showed 0% error for the mesh sizes 0.3 m and less. The larger the grid size, the higher the pore water pressure developed. Outflow rates computed for different grid sizes indicated a maximum error of 120% for a square grid of 1 m and 13.4% for 0.5 m, while the same is less than 5% when the square grid is 0.3 m or smaller after 1 week flow. The computation time required for 1 week flow depended greatly on the total number of mesh openings in the model or grid size and the saturated hydraulic conductivity of the MSW. The higher the saturated hydraulic conductivity, the greater the computation time required for the calculations.
Model Validation

Modeling Mechanical Behavior

Hudson et al. (2004) conducted laboratory-scale experiments to determine the compression (mechanical) behavior of landfilled MSW using the Pitsea compression cell apparatus. This cell can accommodate a waste sample that is 2 m high and 2.5 m in diameter. Features of the test apparatus include hydraulic rams at the top of the cell to simulate overburden pressures in landfills. The test procedure designed by Hudson et al. (2004) involved weighing the test cell after filling it with MSW to a required density and then determining the moisture content of the sample. A vertical load was applied and the settlement observed for two MSW conditions that represent low pore water pressure (pore water pressure less than 20 kPa) and high pore water pressure (pore water pressure between 60 and 70 kPa for saturated MSW). The sample, first saturated by injecting the leachate, was allowed to drain. Observations were made on the moisture content, bulk density, and drainable porosity. Finally, the saturated hydraulic conductivity of the compacted saturated sample was evaluated.

For the model validation study, a model of the Pitsea compression cell was constructed in FLAC 2D (Fig. 9) using the model size parameters of a cell that is 2.5 m high and 2.0 m in diameter. The cell was divided using a 0.3 m square mesh (Fig. 9). All the boundaries were assumed as impermeable, by default. The bottom most grid points are fixed to zero pore water pressure. Since the model simulations include the compression of the MSW under different vertical stresses, the mechanical computations were performed. The deformation in the lateral directions, on either side of the cell, was fixed to zero. The base of the cell was fixed to zero deformation in both lateral and vertical directions. Model simulations were performed for the large strain accumulations in the system.

The initial parameters of the model are summarized in Table 3 and the properties selected represent the specific MSW used by Hudson et al. (2004). The initial pore water pressure was assumed to be zero at all grid points. Flow in unsaturated MSW depends on the unsaturated hydraulic properties of MSW. Therefore, the van Genuchten fitting is used to compute the relative permeability based on the parameters such as the residual saturation, saturated moisture content, matric suction, fitting parameters \(a\), \(b\), \(c\) and \(P_0\). These parameters are selected to represent typical MSW based on the laboratory testing by Stoltz et al. (2012). The coefficient for the pore water pressure increment due to the volumetric strain is assumed as zero to represent flow only calculations as no volumetric strains are computed in the system. The initial porosity of the MSW is computed as 60% (Table 3). Saturated hydraulic conductivity is one of the important parameter that affects the moisture distribution, and therefore, this value is varied between \(10^{-2}\) and \(10^{-5}\) cm/s; the final value of the saturated hydraulic conductivity used was \(4.8 \times 10^{-5}\) cm/s.

The MSW in the experimental cell is assumed to be isotropic and homogeneous. Typical mechanical properties of MSW such as elastic (bulk modulus and shear modulus) and plastic parameters (cohesion, tension, friction angle, and dilation angle) values were selected as summarized in Table 3. The plastic properties for the Mohr-Coulomb model are selected from the published literature presented by Reddy et al. (2009b). The density of the MSW assumed was 1,050 kg/m\(^3\) and the saturated moisture content was assumed to be 41% on volumetric basis.

The MSW in the Pitsea cell was subjected to the normal pressure and a vertical pressure of 34 kPa loading cycle was chosen to represent the initial state for this numerical model. The waste material

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**Table 3. Initial Conditions and Materials Properties Used for Modeling of Mechanical Behavior for the Published Laboratory Study according to Hudson et al. (2004)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Remarks</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual moisture content ((\theta_r)) (%)</td>
<td>20</td>
<td>Laboratory experiments conducted on fresh MSW collected from French bioreactor landfill</td>
<td>Stoltz et al. (2012)</td>
</tr>
<tr>
<td>van Genuchten parameter ((a)) (kPa)</td>
<td>2.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>van Genuchten parameter ((a))</td>
<td>0.318</td>
<td></td>
<td></td>
</tr>
<tr>
<td>van Genuchten parameter ((b))</td>
<td>0.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>van Genuchten parameter ((c))</td>
<td>0.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Porosity (n) (%)</td>
<td>68</td>
<td></td>
<td>Hudson et al. (2004)</td>
</tr>
<tr>
<td>Saturated hydraulic conductivity ((k_{sw})) (cm/s)</td>
<td>(4.8 \times 10^{-5})</td>
<td>Laboratory experiments conducted on fresh MSW collected from French bioreactor landfill</td>
<td>Staub et al. (2010)</td>
</tr>
<tr>
<td>Bulk modulus of MSW (Pa)</td>
<td>(3.5 \times 10^3)</td>
<td>Calibrated</td>
<td>—</td>
</tr>
<tr>
<td>Shear modulus of MSW (Pa)</td>
<td>(0.8 \times 10^3)</td>
<td>Laboratory experiments conducted on MSW collected from Cleanaway Waste Disposal, Essex, U.K.</td>
<td>Hudson et al. (2004)</td>
</tr>
</tbody>
</table>
was modeled to demonstrate elastic, perfectly plastic behavior using Mohr-Coulomb failure criteria. Although the authors cannot fully expect to describe the behavior of municipal solid waste with currently available soil constitutive models (current models do not account for waste decomposition and the resultant changes in stress/strain behavior), the constitutive model chosen for this analysis was adequate for relating the importance of coupled fluid flow and mechanical analysis in bioreactor landfill modeling. Constitutive models, such as Sivakumar Babu et al. (2010) and Machado et al. (2008), have also been used to predict the behavior of MSW. Nevertheless, future modeling studies must aim at predicting MSW behavior with a more comprehensive constitutive model than the Mohr-Coulomb model. For the results comparison, normal stresses of 40, 75, 120, and 240 kPa were applied for 80, 120, 240, and 504 h, respectively, from the top of the model and the responses were noted with respect to the applied stresses under two different pore water pressure conditions. The saturation levels, vertical displacement, volumetric moisture content, and pore water pressure distribution were then computed.

Figs. 10 and 11 indicate that the saturation levels changed slightly as no external flow is allowed to enter the cell. However, it is noteworthy that the saturation level actually tends to increase in the cell as the vertical pressure increases. Furthermore, since the deeper layers are compressed more than the shallow ones, the moisture in the MSW tends to accumulate at the bottom of the cell. The model represents the decrease in volumetric moisture content in the cell due to compression. Since the compacted unit weight in the cell increases under compression, the water present in the voids between the solids will drain out from the bottom of the cell. As shown in Fig. 12, for both high and low pore-water pressure, the initial volumetric moisture content for saturated MSW was high (58 and 48%, respectively) and it reduced to 41 and 25%, respectively, for vertical pressure of 240 kPa (Fig. 12). Based on the results, it is implied that the two-phase model can accurately predict the mechanical behavior of landfilled MSW.

**Coupled Hydromechanical Behavior**

Oliver and Gourc (2005) conducted an experimental study on shredded MSW that used a large-scale compression cell developed initially for testing inert waste such as plastic bottles or tire chips. They upgraded the apparatus to allow hydro-physico-mechanical testing of degradable waste. That apparatus consisted of a rigid square cell 1-m high and 0.98-m wide that is capable of subjecting vertical compression stress up to 130 kPa (equivalent to self-weight of a 15 m thick MSW column) by means of a pneumatic telescopic jack. Leachate generated in the cell was removed from the base to avoid its accumulation. The MSW medium in the sample was raw household waste collected from a sorting/composting unit in Onyx, France, that was composed 55% of organic waste, 14% of plastics, 12% of glass, 7% of textile, 3% of metals, and 9% of other inert materials. The overall gravimetric moisture content of this MSW was determined based on an oven drying method at 105°C for up to 24 h and reported as 35.36%. A total waste mass of 575 kg was...
compacted in the cell in thin layers, with 170 L of leachate added to humidify the MSW. This added moisture represents the Proctor optimum to humidify the waste during compaction. Investigators tested the waste in phases with Phase 0 being compaction of MSW with the optimum moisture. Phase 1 included loading the waste under vertical stress increments of 10 kPa up to 130 kPa that corresponds to increase in the weight of the overlying waste column for the initial 18.75 days. During Phase 1, the water present in the waste mass was allowed to stabilize for 6–10 h followed by a 24 h drawdown to measure the hydraulic conductivity, drainage porosity and corresponding field capacity of the waste material, before the next increment of load was applied. Phase 2 represented the post-construction testing for up to 8.05 months. During Phase 2, the waste temperature was maintained between 32 and 36°C to encourage the microbial activity needed for biodegradation. Investigators measured about 32.5% settlement during Phase 1 and the corresponding primary compression index was noted as 0.28. The secondary compression in Phase 2 yielded about 7.48% of compression under a normal load of 130 kPa and that compression ratio was reported as 0.064. The investigators presented the saturated hydraulic conductivity based on application of Darcy’s law. The obtained values of saturated hydraulic conductivity were within the range of $10^{-5}$ to $10^{-4}$ cm/s for high and low compressive stresses, respectively. The volumetric moisture content and mass moisture content presented considered the evolution of waste height with respect to the applied vertical pressure: the moisture content was relatively stable at about 48–50% at every loading stage.

The Oliver and Gourc (2005) study was replicated in a simulation with a two-phase flow model by coupled flow-mechanical modeling procedure using a model 1-m high and 0.98-m wide in diameter constructed in FLAC 2D. Based on the grid size analysis, a 0.05 m grid is selected as shown in Fig. 13. All the boundaries are assumed impermeable. Since the model simulations include one dimensional compression, the sides are fixed for zero lateral deformation and allowed to deform in the vertical direction only. The cell base is fixed with no deformation allowed in either of the direction. It is important to note that the grid size of 0.05 m (i.e., a relatively finer mesh) was found to be most appropriate in the case...
of Oliver and Gourc (2005) model validation. Since, the flow
distribution in surrounding MSW is dependent upon the gradient
(i.e., the path traveled by the fluid), and smaller the mesh size
is, the path/flow of leachate could be evaluated more accurately.
However, in the case of mechanical-only model (Hudson et al.
2004) validation, fluid flow is ignored, and therefore, a coarser grid
size of 0.3 is appropriate.

The initial model parameters are shown in Table 1. Moreover,
Table 4 lists input parameters that are different from the ones re-
ported in Table 1. Three notable changes in the initial parameters
found in Table 1 were necessary to validate the results of this
published study: the unit weight (750 kg/m$^3$), the bulk and shear
modulus of the MSW (1.5 $\times$ 10$^5$ Pa and 1 $\times$ 10$^5$ Pa), and the
saturated hydraulic conductivity of the MSW (10$^{-4}$ cm/s).

The material properties of the MSW, such as elastic (bulk modu-
lus and shear modulus) and plastic parameters (cohesion, tension,
friction angle and dilation angle) properties are selected from liter-
ature, and calibrated using the study performed by Oliver and
Gourc (2005) for model validation purposes. Moreover, the results
show the efficacy and capability of the proposed model in accu-
rately predicting the behavior of bioreactor landfills. The range
of values used and the final values selected for the elastic properties
of MSW are summarized in Table 4.

Model simulations were carried out for the one-dimensional
compression and are compared with the published experimental
study for the vertical deformation in terms of volumetric strain,
volumetric moisture content, and mass water content. Figs. 14
and 15 represent the vertical deformation contours in the cell
with respect to the applied pressure. A negative sign in the index

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Remarks</th>
<th>Source</th>
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</thead>
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<tr>
<td>Bulk modulus of MSW (Pa)</td>
<td>1.5 $\times$ 10$^5$</td>
<td>Varied between 1 $\times$ 10$^5$ to 4.5 $\times$ 10$^5$</td>
<td>—</td>
</tr>
<tr>
<td>Shear modulus of MSW (Pa)</td>
<td>1.0 $\times$ 10$^5$</td>
<td>Varied between 1 $\times$ 10$^5$ to 2 $\times$ 10$^5$</td>
<td>—</td>
</tr>
<tr>
<td>Dry density of MSW (kg/m$^3$)</td>
<td>750</td>
<td>Laboratory experiments conducted on fresh MSW collected from French bioreactor landfill</td>
<td>Oliver and Gourc (2005)</td>
</tr>
</tbody>
</table>

Fig. 14. Predicted vertical displacement and stress contours in compression cell developed by Oliver and Gourc (2005)
represents compression. The plots give the maximum deformation in the cell and this value was considered when computing change in height with respect to applied pressure to determine total volumetric strain in the cell. The vertical stress distribution in the cell, plotted in Fig. 14, represents the initial stress in the model plus the applied stress on top of the cell. This demonstrates that the maximum vertical stress in the system increases with the applied stress and deformation. The initial stress at the low pressure is 17 kPa, which represents the stress due to the self-weight of MSW plus the applied vertical stress of 10 kPa in the system. Similarly, the maximum stress accumulated in the cell after the final load increment is 135 kPa.

The total volumetric strain in cell was plotted in Fig. 16; the data points represent the experimental observations. The total volumetric strain accumulated in the MSW was more linear for the initial four increments of pressure until 40 kPa was reached. The respective strain accumulated at that increment was 18.3% in the experiment and 19.4% in the model simulations. The load increase continued until 130 kPa was reached and the FLAC model predicted values of volumetric strain close to those presented by Oliver and Gourc (2005). The moisture content in the cell is computed based on the saturation profile obtained after each load increment. Interestingly, since the volumetric moisture content is defined as the ratio of volume of water to the total volume, the volume of water in the cell does not vary significantly; therefore, the saturation in the cell did not vary. The saturation level used for the calculations is the overall saturation value found after each load increment. In the published experimental study, the investigators consider an average value for the saturation after each load increment. Therefore, this study does the same. Since the experiment does not include the addition of moisture in the cell during Phase 1—during compression, the volumetric moisture content in the cell is more or less the

Fig. 15. Predicted vertical displacement and stress contours in compression cell developed by Oliver and Gourc (2005)

Fig. 16. Total volumetric strain and volumetric moisture content in the cell with applied vertical pressure
same throughout the experiment (Fig. 16). On the contrary, since the density of the MSW varies with the applied pressure, the mass of the solids per unit volume increases with the application of pressure and there is no notable increase in the mass of water in the cell. Therefore, the mass moisture content of the MSW is observed to decrease with the applied load increment. A similar trend of the mass moisture content and the volumetric moisture was noted by Oliver and Gourc (2005). Overall, the FLAC model predictions are in good agreement with Oliver and Gourc (2005) and give confidence to applying the mathematical model as a tool for simulating the coupled hydromechanical interaction of MSW in bioreactor landfills.

Summary and Conclusion

A numerical two-phase flow model is presented to demonstrate its ability for efficiently predicting flow-only computations in landfilled MSW (e.g., moisture distribution and generation and distribution of pore fluid pressures), mechanical behavior (stress-strain response) of MSW, and the coupled hydromechanical processes in bioreactor landfills. First, governing equations and mathematical formulations associated with the numerical model are explained in detail. The FLAC is used for numerical implementation of the model using the finite difference method. The model presented assumes that the MSW is unsaturated and leachate and landfill gas are the two immiscible phases. Thereafter, the numerical model was validated in all situations based on comparisons with the findings reported in published mathematical modeling studies conducted in both laboratory and field settings. It is essential to replicate the exact situations appearing in the literature. The appropriate material, boundary, and initial conditions found in the published studies were assumed for use in the two-phase flow model.

The model grid size can influence the model results; therefore, the effects of grid size in flow-only computation are investigated by applying the model to a typical bioreactor landfill cell. Six different square grids of 1, 0.5, 0.3, 0.25, 0.2, and 0.1 m are investigated. Results included the distribution of saturation levels, pore water pressure, outflow collected, and computation time required for 1 week continuous leachate injection at different flow rates and saturated hydraulic conductivity. A typical set of unsaturated hydraulic properties of the MSW was used for this analysis.

Overall, the two-phase flow model represents the unsaturated state of MSW, modeling both the leachate and gas present in the pores, and can be implemented to analyze the flow and hydromechanical response in a bioreactor landfill. The model will prove to be an effective tool capable of providing information necessary for the design of an effective bioreactor landfill model that achieves uniform moisture distribution across the entire landfill based on coupled hydromechanical interactions in landfilled MSW.

Acknowledgments

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