Approach for the use of MSW settlement predictions in the assessment of landfill capacity based on reliability analysis

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Abstract

In the analysis and design of municipal solid waste (MSW) landfills, there are many uncertainties associated with the properties of MSW during and after MSW placement. Several studies are performed involving different laboratory and field tests to understand the complex behavior and properties of MSW, and based on these studies, different models are proposed for the analysis of time dependent settlement response of MSW. For the analysis of MSW settlement, it is very important to account for the variability of model parameters that reflect different processes such as primary compression under loading, mechanical creep and biodegradation. In this paper, regression equations based on response surface method (RSM) are used to represent the complex behavior of MSW using a newly developed constitutive model. An approach to assess landfill capacities and develop landfill closure plans based on prediction of landfill settlements is proposed. The variability associated with model parameters relating to primary compression, mechanical creep and biodegradation are used to examine their influence on MSW settlement using reliability analysis framework and influence of various parameters on the settlement of MSW are estimated through sensitivity analysis.

1. Introduction

Landfilling is still the most common disposal technique for municipal solid waste (MSW) worldwide. In every country, millions of tons of wastes are produced annually and it has become one of the mammoth tasks to dispose it. In the recent times, MSW landfilling has significantly improved and has achieved a stage of well-engineered sanitary landfills in the most developed and developing countries. Evaluation of settlement is one of the critical components in landfill design. The contribution of engineered landfilling requires extensive knowledge of the different processes which occurs simultaneously in MSW during settlement. The settlement of MSW is mainly attributed to: (1) physical and mechanical processes that include the reorientation of particles, movement of the fine materials into larger voids, and collapse of void spaces; (2) chemical processes that include corrosion, combustion and oxidation; (3) dissolution processes that consist of dissolving soluble substances by percolating liquids and then forming leachate; and (4) biological decomposition of organics with time depending on humidity and the amount of organics present in the waste (Reddy et al., 2009a, 2009b). Gourc et al. (2010) presented a one-dimensional biomechanical model to predict the secondary settlement of MSW. The determination of the total secondary settlement was obtained by the addition of two separate parts, the mechanical settlement, due to creep, and the biochemical settlement, due to the degradation of the organic matter. The relative contributions of mechanical and biochemical settlements are also calculated and discussed as a function of waste pre-treatment and operation conditions.

2. Literature review

Due to heterogeneity of MSW, the analysis and design of landfills is complicated. Furthermore, degradation processes in MSW are time dependent phenomena and they continuously change the properties of MSW. In the degradation process, two major mechanisms of biodegradation may occur: aerobic (in the presence of oxygen) and anaerobic (in the absence of oxygen) processes. The production of landfill biogas is a consequence of organic MSW biodegradation. This process is caused by the action of bacteria and other micro-organisms that degrade the organic fraction of MSW in wet conditions. To capture this phenomenon in the prediction of settlement and stress–strain response of MSW, several researchers proposed different models based on the different assumptions (Park and Lee, 1997; Machado et al. 2002, 2008; Marques 2001; Marques et al. 2003; Babu Sivakumar et al. 2010a, 2010b, 2011).
Marques et al. (2003) presented a model to obtain the compression of MSW in terms of primary compression in response to applied load, secondary mechanical creep, and time-dependent biological decomposition. The model performance was assessed using data from the Bandeirantes Landfill, which is a well-documented landfill located in Sao Paulo, Brazil, in which an instrumented test fill was constructed. Machado et al. (2008) presented a constitutive model for MSW based on elasto-plasticity considering that the MSW contains two component groups; the paste and the fibers. The effect of biodegradation is included in the model using a first order decay model to simulate gas generation process through a mass balance approach while the degradation of fibers is related to the decrease of fiber properties with time. Babu Sivakumar et al. (2010a,b) proposed a constitutive model based on the critical state soil mechanics concepts. The model gives the prediction of stress-strain and pore water pressure response, and the predicted results were compared with the experimental results. In addition, the model was used to calculate the time-settlement response of a typical landfill cell. The predicted settlements are compared with the results obtained from the model of Marques et al. (2003) and predictions from both the methods are found to be in the same range. Marques et al. (2003) developed a composite compressibility model that incorporates three mechanisms for one-dimensional compression of MSW, viz. instantaneous response to load, mechanical creep, and biological decomposition. The settlement ($\Delta H$) of the landfill surface at time $t$, due to three mechanisms are represented in term of strains. The total strain ($\varepsilon$) is given by:

$$\varepsilon = \varepsilon_g + \varepsilon_c + \varepsilon_b$$  \hspace{1cm} (1a)

The three terms, $\varepsilon_g$, $\varepsilon_c$, and $\varepsilon_b$, represent strain resulting from instantaneous response to applied load, time-dependent strain due to mechanical creep, and time-dependent strain due to biological decomposition, respectively. The total settlement ($S$), at time, $t$, is given by:

$$\Delta H = \sum_{i=1}^{N} H_i[\varepsilon_{pi} + \varepsilon_{ci}(t) + \varepsilon_{bi}(t)]$$  \hspace{1cm} (1b)

where $N$ is the number of lifts in the landfill and $\Delta H_i$ is the initial thickness of compacted lift.

Babu Sivakumar et al. (2010a) proposed a constitutive model which can be used to determine settlement of MSW landfills based on constitutive modeling approach. In this model, the elastic and plastic behavior as well as mechanical creep and biological decomposition are used to calculate the total volumetric strain of the MSW under loading as follows:

$$d\varepsilon_v = d\varepsilon^e_v + d\varepsilon^c_v + d\varepsilon^b_v$$  \hspace{1cm} (2)

where $d\varepsilon^e_v$, $d\varepsilon^c_v$, $d\varepsilon^b_v$, and $d\varepsilon^i_v$, are the increments of volumetric strain due elastic, plastic, time dependent mechanical creep and biodegradation effects, respectively. The increment in elastic volumetric strain $d\varepsilon^e_v$ can be written as:

$$d\varepsilon^e_v = \frac{-de^e}{1+e} = \frac{\kappa}{1+e} \frac{dp'}{p'}$$  \hspace{1cm} (3a)

And, increment in plastic volumetric strain can be written as

$$d\varepsilon^p_v = \left(\frac{\dot{\varepsilon}}{1+e}\right) \left[\frac{dp'}{p'} + \frac{2\eta d\eta}{M^2 + \eta^2}\right]$$  \hspace{1cm} (3b)

The formulations (3a) and (3b) for increments in volumetric strain due to elastic and plastic are well established in critical state soil mechanics literature (Wood, 1990). In the above equations, $e$ is the void ratio, $p$ is the mean effective stress, $q$ is deviatoric stress, stress ratio is $\eta$ is equal to $q/p$, $\dot{\varepsilon}$ is the slope of normally consolidated line in void ratio ($e$) versus $ln(p)$; $\kappa$ is the slope of swelling line in void ratio versus $ln(p)$; $M$ is slope of critical state line projected to $q'$ versus $p'$ plane; and $d\varepsilon^b_v$ is the increment in plastic volumetric strain.

The mechanical creep is a time dependent phenomenon proposed by Gibson and Lo’s (1961) model, in exponential function is given by

$$\varepsilon^c_v = b\Delta p(1 - e^{-\alpha t})$$  \hspace{1cm} (2)

where $b$ is the coefficient of mechanical creep; $\Delta p$ is the change in mean effective stress, $c$ is the rate constant for mechanical creep; and $\alpha$ is the time since application of the stress increment. The biological degradation is a function of time and is related to the total amount of strain that can occur due to biological decomposition and the rate of degradation. The time dependent biodegradation proposed by Park and Lee (1997), is given by

$$\varepsilon^b_v = E_{bi}(1 - e^{-dt})$$  \hspace{1cm} (3)

where $E_{bi}$ is the total amount of strain that can occur due to biological decomposition; $d$ is the rate constant for biological decomposition; and $t$ is the time since placement of the waste in the landfill.

From Eq. (4), increment in volumetric strain due to creep is written as:

$$d\varepsilon^c_v = c\Delta p e^{-\alpha t} dt$$  \hspace{1cm} (4)

From Eq. (5), increment in volumetric strain due to biodegradation effect is written as:

$$d\varepsilon^b_v = dE_{bi} e^{-dt} dt$$  \hspace{1cm} (5)

In the present case $t$ time since application of the stress increment and $t'$ time since placement of the waste in the landfill are considered equal to $t$. It is necessary to note that this is a simplification as both the times are different for the above processes.

Using Eqs. (3a), (3b), (6), and (7), and substituting in Eq. (2), total increment in strain is given by

$$d\varepsilon_v = \kappa \frac{dp'}{1+e} + \left(\frac{\dot{\varepsilon}}{1+e}\right) \left[\frac{dp'}{p'} + \frac{2\eta d\eta}{M^2 + \eta^2}\right] + c\Delta p e^{-\alpha t} dt + dE_{bi} e^{-dt} dt$$  \hspace{1cm} (6)

The model parameters used for the prediction of MSW settlement are similar to the Marques et al. (2003). Calculation procedure of settlement response of MSW using above equations is given in Babu Sivakumar et al. (2011). The procedure for calculating the settlements is quite involved and it is useful to convert the procedure to a simple mathematical form in the form of explicit relationship relating the parameters as variables to settlement. An effort is made in this study using the response surface method (RSM) in this direction. The advantage of the Response surface method is that it can consider variability of the parameters influencing the settlement predictions. The developed relationship is also useful for reliability analysis.

3. Variability of MSW parameters

Settlement models of Marques et al. (2003) and Babu Sivakumar et al. (2010a,b) use parameters such as compressibility index ($\lambda$), coefficient of mechanical creep ($b$), creep constant ($c$), biodegradation constant ($E_{bi}$) and rate of biodegradation ($d$). All these parameters are highly variable due to heterogeneity of MSW. For the engineering design of landfill, these variables are design parameters and their variability plays vital role in design. Literature review indicates that the influence of all these parameters and their variations have significant effects on prediction of MSW settlement. Based on experimental and field observations various
researchers reported different range of values and percentage of coefficient of variations (COV) for the parameters. For example, Sowers (1973) reported that the compression index ($c_0$) is related to the initial void ratio ($e_0$) and varies between 0.15 $e_0$ and 0.55 $e_0$ and the value of secondary compression index ($c_s$) varies between 0.03 $e_0$ and 0.09 $e_0$. The upper limit corresponds to MSW containing large quantities of food waste and high decomposable materials. Results of Gabr and Valero (1995) indicated $c_s$ values varying from 0.4 to 0.9, and $c_s$ values varying from 0.03 to 0.009 for the initial void ratios ($e_0$) in the range of approximately 1.0–3.0. Machado et al. (2002) obtained the values of primary compression index which varied between 0.52 and 0.92. Marques et al. (2003) also reported range and COV values for coefficient of mechanical creep ($b$) and rate for mechanical creep ($c$) constant as 0.000292–0.000726 and COV of 17.7% and 0.000969–0.00255 and COV of 26.9% respectively. The time dependent strain due to biodegradation is expressed by an equation which uses $E_{bi}$, the parameter related to total amount of strain that can occur due to biodegradation, and $d$, the rate constant for biological decomposition. Biodegradation constant depends upon the organic content present in MSW. Marques et al. (2003) gave typical range of $E_{bi}$ varying from 0.131 to 0.214 with a COV of 12.7% and biodegradation rate constant $d$ varying from 0.000677 to 0.00257 with a COV of 42.3%. Foye and Zhao (2011) used random field model to analyze differential settlement of existing landfills. They used $c_s$ values 0.22 and 0.29 with COV of 36% and $E_{bi}$ time dependent strain due to biodegradation equal to 0.03724 and rate constant due to biodegradation ($d$) equal to 0.0007516. The values of all the above parameters are variable; and their values depend upon the various factors like site conditions, initial moisture content, and quantity of biodegradable material present in the existing MSW, etc. Therefore, it is very important to perform settlement analysis of MSW considering the variability of design parameters. The importance of variability in geotechnical and geo-environmental designs is documented in literature (Babu Sivakumar 1998, Duncan, 2000, Baecher and Christian, 2003, Babu Sivakumar et al. 2010c; Foye and Zhao 2011). The effect of variability is examined in terms of its influence on reliability index or probability of failure.

4. Reliability analysis

4.1. Need for the reliability analysis

Important concerns in the landfill engineering are the estimation of landfill capacity and the development of closure plans based on predicted measured settlements. In this context, it is very worthwhile to know the probability of a particular height/capacity of the landfill/cell reaching an expected value as the parameters for the estimation of settlement are random variables. The objective of the present study is to demonstrate the influence of variability in model input parameters on the prediction of MSW settlement. To capture the effect of variability, response surface method (RSM) is used in conjunction with settlement model to develop multi-linear relationships among all the design variables. Based on these generated equations for the different percentage of COVs, reliability analysis is conducted and reliability index is evaluated for a typical landfill cell.

Reliability is defined as probability of safety of a system under given environment and loading conditions. Reliability and probability of failure are mutually exclusive and the evaluation of reliability or probability of safety of a system is based on specific performance criterion. For example, the specific criterion can be defined in terms of terms probability of a factor of safety being more than unity or the predicted settlements being less than the permissible values. It is assessed in terms of probability of failure and reliability index ($\beta$) values. The United States Army Corps of Engineers (USACE, 1997) provides guidelines for reliability based designs. The guidelines indicate that the reliability index for infrastructure projects can vary from 0 to 5 with the values 3 (corresponding to a probability of failure of 0.001) and above are considered acceptable. The performance function is defined in terms of the basic variables $X_i$ and the functional relationship among them. Mathematically, this relationship can be described as

$$Z = g(X_1, X_2, \ldots, X_n)$$

The failure surface or limit state can be defined as $Z = 0$, this is the boundary between the safe i.e. $Z > 0$ and unsafe region $Z < 0$ in the design parameters. Using Eq. (9) the failure occurs when $Z < 0$. Therefore, the probability of failure, $P_f$ is given by the integral

$$P_f = \int_{[g(X) \leq 0]} \int_{[f_1(x_1, x_2, \ldots, x_n) \cdot dx_1 \ldots dx_n]}$$

where $X = [X_1, X_2, \ldots, X_n]^T$ ($T$ is the transpose of matrix of random variables and $f_1(x_1, x_2, \ldots, x_n)$ denotes the joint probability density function of $X$.

In the literature (Baecher and Christian, 2003), various methods are available for the calculation of reliability index, such as first order second moment method (FOSM), Hosfer–Lind method (AFOSM), first order reliability method (FORM), second order reliability method (SORM), Monte Carlo simulation (MCS), and point estimation method (PEM). The performance function $g(X)$ for a linear performance function and the reliability index $\beta$ for the independent variables in $n$-dimensional space are given as:

$$g_i(X) = c_0 + \sum_{i=1}^{n} c_i X_i$$

$$\mu_g = c_0 + c_1 \mu_{x_1} + c_2 \mu_{x_2} + \ldots + c_n \mu_{x_n}$$

$$\sigma_g^2 = \sum_{i=1}^{n} c_i^2 \sigma_{x_i}^2$$

$$\beta = \frac{\mu_g}{\sigma_g}$$

where $\mu_g$ and $\sigma_g$ are the mean and standard deviation of the approximate limit-state function and the terms $c_i$ denote the regression constants for the performance function.

4.2. Response surface method

RSM is a collection of statistical and mathematical techniques useful for developing, improving, and optimizing processes (Myers and Montgomery, 2002). For the practical application of response surface method (RSM), it is necessary to develop an approximating model for the true response surface. A first order multi-linear response surface model, obtained based on regression analysis is given by

$$y_i = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_n x_n + \varepsilon$$

Here, $y_i$ is the observed settlement of MSW, the term “linear” is used because Eq. (15) is a linear function of the unknown parameters: $\beta_1, \beta_2, \ldots, \beta_n$, which are called regression coefficients, and $x_1, x_2, x_3, \ldots, x_n$ which are coded variables that are usually defined to be dimensionless with mean zero and standard deviation. The natural variables ($b, c, d, E_{bi}, \lambda$) are converted into coded variable using the following relationship:

$$x_i = \frac{z_i - \left[\max(z_i) + \min(z_i)\right]/2}{\left[\max(z_i) - \min(z_i)\right]/2}$$


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where \(\xi\) represents natural variables with maximum and minimum values.

RSM analysis is performed using single replicate 2\(^n\) factorial design to fit first order linear regression model, where \(n\) is the total number of input variables involved in the analysis and corresponding to these variables the number of sample points required is 2\(^n\). The method of least squares is used to estimate the regression coefficients in a multiple linear regression model. Myers and Montgomery (2002) presented the multilinear regression expression in the form of matrix:

\[ y = X\beta + e \]  \hspace{1cm} (16a)

And the least squares estimators \(\beta\) are given by,

\[ \beta = (X'X)^{-1}X'y \]  \hspace{1cm} (16b)

4.3. Residual analysis

Analysis of residuals is conducted to examine if the regression equations provide an adequate approximation to the true system and verify that none of the least square regression assumptions is violated. The residuals from the first order response surface method, defined by \(e_i = y_i - \hat{y}_i\), \(i = 1, 2, 3, \ldots, n\) (where, \(y_i\) is the value obtained using the procedure based on constitutive model and \(\hat{y}_i\) is the predicted value from regression model) play an important role in judging model adequacy. If the residual plot falls along a straight line, then the normality assumption is satisfied. Further, computed values of the coefficient of regression \(R^2\) and the adjusted coefficient of regression \(R^2_{adj}\) also give description of adequacy of fitted model. Regression coefficients \(R^2\) and \(R^2_{adj}\) are calculated from the equation given below:

\[ R^2 = \frac{b^2 \sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2}{\sum_{i=1}^{n} y_i^2} \]  \hspace{1cm} (17a)

\[ R^2_{adj} = 1 - \frac{n - 1}{n - p}(1 - R^2) \]  \hspace{1cm} (17b)

where, \(b\) is the transpose of the Eq. (16b) and \(y_i\) is the observed value of settlement, \(n\) is total number of sample points and \(p\) is the number of variables. The value of \(R^2\) varies from 0 to 1 and value close to 1 indicates that the most of the variability of \(y_i\) (settlement) obtained by using the regressor (independent) variables \(x_1, x_2, x_3\) in the model is explained by the regression equations. It may be noted that the use of the procedure to calculate settlement based on constitutive modeling is quite involved where as the use of regression equations is straightforward. However, it may be noted that the regression equations are specific to the boundary conditions, mean values of properties and COVs and hence cannot be generalized, but are very useful in reliability calculations as demonstrated below with regard to the estimation of probability of settlement of MSW and its use in landfill closure.

4.4. Reliability index formulation

Fig. 1 illustrates a typical landfill cell with the height of 30 m, consisting of 10 layers of 3 m thick. After completing the filling of the landfill, a final cover system is assumed to be constructed that consists of composite liner, compacted clay and geomembrane overlain by a sand drainage layer and then a vegetative cover soil layer. The cover system also has surcharge of 40.22 kN/m\(^2\). The thickness and unit weights materials of different layers are also given in the figure. It is required to estimate the ultimate settlement in 30 years as well as its variability, given that the input parameters for evaluation of settlement are random variables.

![Fig. 1. MSW landfill scenario for estimation of settlement versus time calculations.](image)

The expected ultimate settlement is also a random variable and it is often useful in landfill closure operations, to know the probability of ultimate settlement reaching a particular value less than the expected value. This helps in planning and developing closure plans of landfills so that the additional space that becomes available can be properly utilized and the required waste quantities can be collected.

5. Results and discussion

5.1. Regression equations

For the analysis, the maximum and minimum values are assigned based on COVs. The mean and COV values of variables used in the present study are given in Table 1 and are adopted from Marques et al. (2003). The values used are marginally different from the quoted values. Using 2\(^n\) factorial designs, sample points are generated for different coefficients of variation (COVs) and corresponding to these samples, values of settlement are calculated. For example in the present case, five variables are considered, hence \(n\) is equal to 5, and the number of sample points required is 32. These 32 sample points are generated using “+" and “−" notation to represent the high and low levels of each factor, the thirty-two runs in the 2\(^5\) design are conducted and results obtained. The sample points corresponding to the five parameters are generated and corresponding settlements are calculated using the approach suggested based on constitutive model by Babu Sivakumar et al. (2011). The response surface equations for 10% and 20% COVs and general values of COV of parameters obtained from multi-linear regression are as follows:

\[ y_{f10\%} = 21.62\lambda + 5067.53b + 0.0050c + 0.20d + 0.037E_{Edg} + 4.39 \]  \hspace{1cm} (18a)

\[ y_{f20\%} = 21.93\lambda + 5069.25b + 0.0036c + 0.37d + 0.020E_{Edg} + 4.38 \]  \hspace{1cm} (18b)

\[ y_{fav} = 20.42\lambda + 5130b + 12.14c + 13.40d + 0.030E_{Edg} + 4.36 \]  \hspace{1cm} (18c)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>COV%</th>
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</thead>
<tbody>
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<td>(\lambda)</td>
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<td>(b)</td>
<td>5.27E-04</td>
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<td>(c)</td>
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<td>(d)</td>
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<td>(E_{Edg})</td>
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<tr>
<td>(F)</td>
<td>12.99</td>
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</table>
The adequacy of proposed multilinear response surface equations are evaluated from residual analysis and coefficients of regression ($R^2$).

5.2. Analysis of residuals and regression

In order to examine the adequacy of proposed response surface equations, normal probability versus residuals plot is obtained. Settlements obtained from response surface equations and using the procedure from constitutive model are compared and it is observed that the fitted and observed settlements are in good agreement. Table 2 presents typical results of the settlements obtained from both the approaches along with the corresponding residuals for 10% COV. It can be observed that the range of values from the approach based on constitutive model vary from 10.42 to 8.19 m and the values from predictions using regression equation for 10% COV are quite close and comparable. In addition, a check of the normality assumption is made by the construction of a normal probability plot of the residuals and typical results for COV of 20% are presented in Fig. 2. It is observed that the residual plot falls along a straight line indicating that the normality assumption is satisfied. In addition, computed values of coefficients of regression ($R^2$) and adjusted $R^2$ also give description of adequacy of fitted model. For a good model, values of $R^2$ and $R^2$ should be close to 1. The calculated values are in the range of 0.997 indicating the adequacy of the regression equations.

5.3. Reliability analysis

Calculations performed using the parameters value given in Table 1 from constitutive model as well as response surface equations given by (18) presented in the earlier section gives expected value of ultimate settlement 8 m corresponding to 30 years. However, it is necessary to know the variability associated with this value, knowing that the input parameters are random variables. Under these conditions, it is useful to evaluate the probability of obtaining a certain value less than the expected values ($y_f$) or a value 5 m or 6 m or 7 m which are less than the expected ultimate settlement. For example, the probability of obtaining ($y_f$) (6 m) as ultimate settlement for the given loading conditions is given by:

$$P_{6m} = P(y_f \leq 6)$$

(19)

where $y_f$ is likely settlement value.

As the probability of obtaining 6 m ultimate settlement is required, the performance function of all the input variables have 10% COV associated with them is expressed as:

$$21.62\lambda + 5067.53\beta + 0.005c + 0.20d + 0.037Edg + 4.39 - 6$$

Or

$$21.92\lambda + 5062.3b + 0.005c + 2.33d + 36Edg - 1.61$$

(20a)

Similarly, performance function for 20% COVs is given by

$$21.93\lambda + 5069.25b + 0.0036c + 0.37d + 0.020Edg + 4.38 - 6$$

Or

$$21.93\lambda + 5069.25b + 0.0036c + 0.37d + 0.020Edg - 1.62$$

(20b)

Eq. (20) represents performance functions in the form of multilinear response surface equations that includes all the contributing variables used for the prediction of MSW settlement in the form of natural variables ($\lambda, \beta, c, d, Edg$).

Knowing approximated mean ($\mu_k$) and standard deviation ($\sigma_k$), reliability index is calculated using relation given by $\beta = \mu_k/\sigma_k$. Table 3 presents the summary of variation of reliability index values for 10% and 20% COVs of input parameters. It is observed from Table 3 that the reliability index is inversely proportional to the COV of design variables and reliability index ($\beta$) decreases with increase in percentage of COVs of the design variables.

It can be stated that the probability of landfill settlement being 6 m depends on the variability of the parameters that are used to

<table>
<thead>
<tr>
<th>No.</th>
<th>Predicted values ($y_i$)</th>
<th>Observed values ($y_i$)</th>
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Fig. 2. Normal probability plot of the residuals for COV 20%.
calculate the settlement. Additional results presented in Table 4, clearly point out the effect of variability on the reliability index corresponding to different expected levels of filling varying from 4 to 8 m. It is observed that for 10% COV the reliability index corresponding to 6 m ultimate settlement is obtained as 7.11, whereas for 20% COV corresponding to 6 m reliability index is calculated as 3.56. If the variability is less, the probability of settlement being 6 m is high and vice versa.

Hence good quality control in compaction and placement of wastes, testing of MSW samples and estimation of parameters is essential and likely to lead to low values of variability associated with parameters and ultimately lead to better reliability estimates of reliability associated with settlement predictions. Table 5 presents the mean values, standard deviations, and COVs of settlement corresponding to different COVs of input parameters. It can be noted that COV of settlement increases with increase in COV of the input variables. This suggests that the standard deviations are high if the variability of parameters is high.

6. Concluding remarks

In this paper, an approach to use landfill settlements and develop landfill closure plans based on the variability of design parameters is suggested. Considering five design variables in the evaluation of MSW settlement using the response surface method (RSM) and the constitutive model and based on the multi-linear equations are derived from RSM, the performance functions are defined and reliability index values are calculated. However, it may be noted that the regression equations are specific to the boundary conditions, mean values of properties and COVs and hence cannot be generalized, but are very useful in reliability calculations. The proposed approach to develop landfill closure plans based on the variability of parameters and reliability analysis is illustrated with typical example. The following are the major conclusions emerge from this study:

1. Response surface equations are developed considering five variables for different percentage of COVs gives estimated settlement which is close to the predicted settlement from proposed constitutive model.
2. Reliability index is inversely proportional to the variance of the design parameters (COV). As a result, highly variable MSW results in poor estimates of reliability of landfill settlements and as well as variability.
3. It is useful to collect the data on variability of landfill material properties and also examine their implications in landfill engineering design and closure operations.

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References


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<th>Table 4</th>
<th>Variation of reliability index with respect to filling capacity of landfill.</th>
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<th>Mean, standard deviation and coefficients of variation of settlement.</th>
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