Prediction of Long-Term Municipal Solid Waste Landfill Settlement Using Constitutive Model

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Abstract: This paper proposes a generalized constitutive model for municipal solid waste, based on the framework derived from critical state concepts and incorporating the effects of mechanical creep and time-dependent biodegradation, to predict total landfill compression under incremental loading and with time. The model parameters are calculated based on laboratory one-dimensional compression and triaxial compression tests, and data available from published literature. To illustrate the applicability of the proposed model, settlements due to incremental loading of waste with time are predicted for typical landfill conditions. The predicted settlement results using the proposed model are compared with the predicted settlement results using 14 different reported models. It is shown that the predicted settlements can vary significantly depending on the model used and the parameter values selected. The proposed model predicts the total settlement in a range similar to the reported models that consider all three components (mechanical, creep, and biodegradation) of the settlement. Further validation of the model is required based on field measured settlement at different landfill sites.

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Introduction

Landfilling is a common method of disposal of municipal solid waste (MSW). Landfills are engineered structures consisting of bottom liners, leachate collection and removal systems, and final covers. MSW settles under its own weight and as external loads are placed on the landfill. External loads include daily soil cover, additional waste layers, final cover, and facilities such as buildings and roads. MSW settlement is mainly attributed to: (1) physical and mechanical processes that include the reorientation of particles, movement of the fine materials into larger voids, and collapse of void spaces; (2) chemical processes that include corrosion, combustion and oxidation; (3) dissolution processes that consist of dissolving soluble substances by percolating liquids and then forming leachate; and (4) biological decomposition of organics with time depending on humidity and the amount of organics present in the waste. Significant settlement occurs during and shortly after waste placement due to physical and mechanical processes, which is often referred to as primary settlement. Substantial additional settlement occurs at a slower rate over an extended period of time due to chemical and biological processes, which is often referred to as secondary settlement (Sowers 1973).

It is necessary to predict the long-term settlement of MSW for the final cover design as well as end-use facility design (e.g., recreational facilities, industrial/commercial facilities). In addition, estimation of settlement is needed to assess the stability of leachate and gas collection pipes, drainage systems, landfill storage capacity, and the overall landfill operating costs. Excessive settlement may cause fracture in the cover system and may also cause damage to the drainage and leachate/gas collection pipes.

Several researchers have proposed models to predict the settlement of MSW considering different combinations of landfill conditions. Many of these methods are based on soil mechanics-based consolidation and secondary compression processes and/or empirical equations. The importance of settlement models based on the mechanical behavior of MSW under loading has been recognized recently and efforts are being made to develop general models to describe the response of MSW under stress with time.

This paper provides a summary of reported settlement models followed by a description of the development of a generalized constitutive model, based on critical state concepts and accounting for mechanical creep and time-dependent biodegradation, to calculate total landfill compression under loading with time. For hypothetical landfill conditions, the model predictions are compared with the predictions based on the reported models. The advantages of using the developed model to evaluate the stability of landfill under long-term conditions as compared to the published models are also discussed.

Published Settlement Models

Several models are reported in the published literature to predict MSW landfill settlement. These models may be grouped as: (1) soil mechanics-based models; (2) empirical models; (3) rheologi-
Soil Mechanics-Based Models

Sowers (1973) first used the basic soil mechanics-based model of consolidation to estimate the settlement of MSW. The long-term compression associated with creep and biodegradation phenomena is expressed in terms of the secondary compression index \( C_a \) in which a decrease in the void ratio during the secondary compression is related to the time elapsed between the initial time \( t_1 \) and the final time \( t_2 \). The model can be expressed as

\[
\Delta H = HC_a \left( \log \left( \frac{\sigma_0 + \Delta \sigma}{\sigma_0} \right) + HC_a \log \left( \frac{t_2}{t_1} \right) \right)
\]

where \( \Delta H = \) settlement due to primary and secondary consolidation; \( H = \) initial thickness of the waste layer; \( C_a = \) primary compression ratio; \( \sigma_0 = \) existing overburden pressure acting at midlevel of the layer; \( \Delta \sigma = \) increment of overburden pressure acting at midlevel of the layer from the construction of an additional layer; \( C_c = \) secondary compression index; \( t_1 = \) time of initial compression; and \( t_2 = \) ending time period for which long-term settlement of layer is desired. The values of compression indices \( C_a \) and \( C_c \) for MSW are reported to range from 0.163 to 0.205 and 0.015 to 0.350, respectively.

Bjerrum and Edgers (1990) subdivided the secondary compression into two subphases, through the adjustment of two straight lines, and introduced the intermediate coefficient of secondary compression \( C_{a1} \) and a final coefficient of secondary compression \( C_{a2} \). The settlement model can be expressed as

\[
\Delta H = HC_c \left( \log \left( \frac{\sigma_0 + \Delta \sigma}{\sigma_0} \right) + HC_{a1} \log \left( \frac{t_1}{t_2} \right) + HC_{a2} \log \left( \frac{t_3}{t_2} \right) \right)
\]

where \( \Delta H = \) settlement due to primary and secondary consolidation; \( H = \) initial thickness of the waste layer; \( C_c = \) primary compression ratio; \( \sigma_0 = \) existing overburden pressure acting at midlevel of the layer; \( \Delta \sigma = \) increment of overburden pressure acting at midlevel of the layer from the construction of an additional layer; \( C_{a1} = \) intermediate secondary compression ratio; \( C_{a2} = \) long-term/final secondary compression ratio; \( t_1 = \) time for initial compression; \( t_2 = \) time for intermediate secondary compression; and \( t_3 = \) time for total period of time considered in modeling. Typical parameter values are reported to be \( C_c = 0.205 \), \( C_{a1} = 0.035 \), \( C_{a2} = 0.215 \); \( t_1 = 1 \) to 25 days; and \( t_2 = 200 \) days.

Hossain and Gabr (2005) modeled long-term settlement with three terms as given in Eq. (3)

\[
\frac{\Delta H}{H} = C_a \log \left( \frac{t_2}{t_1} \right) + C_{b1} \log \left( \frac{t_3}{t_2} \right) + C_{a2} \log \left( \frac{t_4}{t_3} \right)
\]

where \( C_a = \) compression index, which is a function of stress level and degree of decomposition (\( \sim 0.03 \)); \( t_1 = \) time for completion of initial compression (\( \sim 10 \)–15 days); \( t_2 = \) time duration for which compression is to be evaluated (\( \sim 100 \) to 2,000 days); \( C_{b1} = \) biodegradation index (\( \sim 0.19 \)); \( t_3 = \) time for completion of biological compression (\( \sim 3,500 \) days); \( C_{a2} = \) creep index; and \( t_4 = \) time for the creep at the end of biological degradation. The mechanical compression under applied stress and/or the pressure due to self-weight were not included. As biodegradation occurs, the organic solid mass is converted to gas and the void ratio increases with a subsequent increase in waste settlement. The model developed was based on the results of experimental program. The degree of decomposition was characterized from gas generation rates and the cellulose plus hemicellulose to lignin ratio. The time factors, \( t_1, t_2, t_3, \) and \( t_4, \) for the compressibility were determined from the gas production curve and utilized for model development.

Empirical Models

The empirical models attempt to simulate general waste behavior by adjusting the empirical parameters, which are site specific. Common mathematical functions employed are the logarithmic function, the power creep function, and the hyperbolic function. The logarithmic function is expressed as (Yen and Scanlon 1975)

\[
\Delta H = H_f \left[ \alpha + \beta \log \left( t - \frac{t_2}{2} \right) \right]
\]

where \( \Delta H = \) settlement rate; \( H_f = \) initial height of the landfill; \( \alpha = \) fitting parameter \((=0.00095H_f + 0.00969)\); \( \beta = \) fitting parameter \((=0.0035H_f + 0.00501)\); \( t = \) time since beginning of filling; and \( t_2 = \) construction period.

The power creep model is a simple relation for time-dependent deformation under constant stress and it is given by (Edil et al. 1990)

\[
\Delta H = H_0 \Delta \sigma M' \left( \frac{t}{t_0} \right)^{N'}
\]

where \( \Delta H = \) settlement; \( H_0 = \) initial height of waste; \( \Delta \sigma = \) compressive stress depending upon waste height, density, and external loading; \( M' = \) reference compressibility \((=1.6 \times 10^{-5} \text{ to } \sim 5.8 \times 10^{-5} \text{ kPa})\); \( N' = \) rate of compression \((=0.50 \text{ to } 0.67)\); \( t = \) time since load application; and \( t_0 = \) reference time typically introduced to make time dimensionless (usually taken as 1 day) (El-Fadel et al. 1999). The parameter \( M' \) is site specific and \( N' \) is the rate of compression, which varies with respect to age and placement conditions of the waste. However, the variability of \( N' \) is less than that of \( M' \) (Edil et al. 1990).

Ling et al. (1998) proposed the following hyperbolic equation to compute settlement at a given time if the ultimate settlement of the landfill is known:

\[
\Delta H = \frac{t}{(1/p_0) + (t/S_{ah})}
\]

where \( \Delta H = \) settlement; \( p_0 = \) initial rate of settlement \((=0.001 \text{ m/day})\); \( S_{ah} = \) ultimate settlement; and \( t = \) time since load application.

Cournoulos and Koryalos (1997) proposed an attenuation equation, which is based on the proposition that landfill settlements can be approximated by a straight line, as a function of the logarithm of time. The main advantage of this model is that data from different points on the landfill with different characteristics can be grouped and compared. The model can be expressed as

\[
Y = \frac{d(\Delta H/H)}{dt} = 0.434C_a \frac{t}{t_+ + (t/2)}
\]

where \( Y = \) vertical strain rate (expressed in percent/month or percent/year); \( C_a = \) coefficient of secondary compression \((=0.02 \text{ to } 0.25)\); \( t = \) time elapsed in month or year; and \( t_+ = \) filling time usually assumed as 1 month. It must be noted, however, that the accuracy of \( Y \) depends on the accuracy of \( C_a \).
**Rheological Model**

Gibson and Lo (1961) proposed a model that is applicable to peaty soils. This model is used by Edil et al. (1990) to predict long-term total settlement of MSW. The model uses an analogy that represents primary compression and secondary compression in which a compression of a spring expresses immediate compression and a combination of piston and spring expresses the slow deformation. The model can be expressed as

\[
\frac{\Delta H}{H_0} = \Delta \sigma_a + \Delta \sigma_b (1 - e^{-k/b})
\]  

where \(\Delta H\) = settlement; \(H_0\) = initial height of waste; \(\Delta \sigma_a\) = compressive stress depending upon waste height, density, and external loading; \(\Delta \sigma_b\) = secondary compressive stress; \(a\) = primary compressibility parameter (1.0 \times 10^{-3} to 8.0 \times 10^{-5} kPa); \(b\) = secondary compressibility parameter (2.0 \times 10^{-3} to 1.6 \times 10^{-2} kPa); \(k\) = rate of secondary compression (1.4 \times 10^{-4} to 9.0 \times 10^{-3}/day); and \(t\) = time since load application (El-Fadel et al. 1999).

**Settlement Models Incorporating Biodegradation**

Park and Lee (1997) proposed a settlement model that considers time-dependent biodegradation of waste. The settlement rate is assumed to be the same as that of solids solubilized. Solubilization of organic materials is generally expressed using first-order kinetics. However, the determination of the kinetic coefficients or the hydrolysis constants as well as their variation with environmental conditions is difficult. The settlement model can be expressed as

\[
e(t)_{\text{sec}} = C_a \log \left( \frac{t_2}{t_1} \right)
\]  

where \(C_a\) = rate of secondary compression; \(k\) = first order decomposition rate constant/time (2.37 to 1.35/year); \(t_{\text{sec}}\) = total amount of compression that will occur due to decomposition of biodegradable waste (7.2–6.1%). The summation of both the terms gives the total compressive strain.

Hettiarachchi et al. (2009) also developed a settlement model assuming that the settlement due to biodegradation follows the first-order reaction kinetics. The total settlement is expressed as a combination of mechanical compression and \((\Delta H)_b\) and biodegradation-induced settlement or \((\Delta H)_b\). The model is expressed as

\[
(\Delta H)_b = H_i \left[ \frac{M_d}{\rho_w} \sum_{j=1}^{4} \frac{f_{sj}}{G_{sj}} (1 - \exp^{-\lambda_j}) \right]
\]  

\[
(\Delta H)_m = H_i C^* \log \left( \frac{\sigma' + \Delta \sigma'_{\text{bi}}}{\sigma'} \right)
\]  

\[
H_f = H_i - (\Delta H)_b - (\Delta H)_m
\]  

where \(C^*\) = compressibility parameter (0.174–0.205); \(\sigma'\) = effective stress; \(\Delta \sigma'\) = difference in effective stress; \(f_{sj}\) = initial solids fraction for each waste group (0.15–0.35); \(\rho_w\) = density of water; \(G_{sj}\) = specific gravity of jth group of waste solids (1–3); and \(\lambda_j\) = first-order kinetic constant for the jth group (0–0.001/day).

Marques (2001) developed a composite rheological model to account for primary and secondary compression mechanisms, governed by rheological parameters that also accounts for waste degradation. The primary compression formulation is introduced as an “immediate compression,” which is independent of time, based on the observation that the respective process is linear for curves of void ratio as a function of the logarithm of the applied stress. The model is represented by

\[
\frac{\Delta H}{H_i} = C'_c \log \left( \frac{\sigma_0 + \Delta \sigma}{\sigma_0} \right) + \Delta \sigma \cdot b \cdot (1 - e^{-c't}) + E_{d_b} \cdot (1 - e^{-d't})
\]  

where \((\Delta H)/H_i\) = settlement; \(H_i\) = initial height of waste; \(C'_c\) = primary compression ratio; \(b\) = coefficient of secondary mechanical compression; \(c\) = secondary mechanical compression rate; \(E_{d_b}\) = total compression due to waste degradation; \(d\) = secondary biological compression rate; \(t\) = time elapsed since loading application; and \(t'\) = time elapsed since waste disposal.

Marques et al. (2003) further developed a composite compressibility model that incorporates three mechanisms for one-dimensional compression of MSW: instantaneous response to load, mechanical creep, and biological decomposition. MSW is usually placed in lifts in landfills over a period of years. In implementing the composite model, waste placement is idealized as progressing in a series of lifts. The thickness of the lifts may be set equal to the compacted thickness of the daily cells. After all lifts have been placed, the settlement \((\Delta H)\) of the landfill surface at time \(t\) is determined. The total strain is given by

\[
e = e_p + e_c + e_b
\]  

The three terms, \(e_p\), \(e_c\), and \(e_b\), represent strain resulting from instantaneous response to applied load, time-dependent strain due to mechanical creep, and time-dependent strain due to biological decomposition. The total settlement is given by

\[
(\Delta H) = \sum_{i=1}^{N} \left[ e_p(i) + e_c(i) + e_b(i) \right]
\]  

where \(N\) = number of lifts in the landfill; \(H_i\) = initial thickness of compacted lift \(i\); \(e_p\) = strain in lift \(i\) resulting from instantaneous response to loading from overlying lifts; \(e_c\) = strain at time \(t\) in lift \(i\) due to mechanical creep associated with the stresses from self-weight and the weight of overlying lifts; and \(e_b\) = strain at time \(t\) in lift \(i\) due to biological decomposition of lift \(i\). The strains are given by

\[
e_p(i) = C'_p \log \left( \frac{1}{2} \gamma_i H_i + \sum_{j=1}^{N} \Delta \sigma_{ij} \right)
\]  

\[
e_c(i) = b \left[ 1 - \gamma_i H_i t \cdot (1 - e^{-c'i}) \right] + \sum_{j=1}^{N} \Delta \sigma_{ij} (1 - e^{-c'i})
\]  

\[
e_b(i) = E_{D_b} (1 - e^{-d' t i})
\]  

where \(\gamma_i\) = unit weight of lift \(i\) is a composite value representing the weighted average of the compacted waste and daily cover in lift \(i\). Because \(t\) is time after all lifts of the landfill have been placed, \(i = i + 1\) on lift \(i\) for \(j > i\); and \(t_i\) and \(t_i\) are times at which lifts \(i\) and \(j\), respectively, were placed. Boussinesq theory is used to determine change in vertical stress. \(C'_p\) = compression ratio (0.0732 to 0.320); \(E_{D_b}\) = total amount of strain that can occur due to biological decomposition.
The decomposition settlement during filling is given by

\[ U_d = \frac{0.87HM'}{t_c C_c'} \]  

(13a)

where \( U_d \) = primary settlement of waste for design thickness; \( H \) = design thickness; \( C_c' \) = compression index in terms of vertical strain (0.10 to 0.50); \( m' \) = rate of filling (thickness increase per unit time); and \( t_c \) = time of completion of filling. 

At time \( t \) (months) and closure time \( t_c \), the estimated creep settlement is calculated using

\[ U_c = 0.435C_a'm'[1 + \{t ln(i)\} - i] \text{(creep settlement at time } \ t) \]  

(13b)

\[ U_{cre} = 0.435C_a'm'[1 + \{i ln(i)\} - i_c] \times \text{(creep settlement at closure time } \ t_{c'}) \]  

(13c)

where \( C_a' \) = modified coefficient of secondary compression in terms of strain (0.02–0.32). Assuming, filling stops at time \( t_c \), the additional (post filling) creep settlement is

\[ \Delta U_j = 0.435HC_a'\ln \left( \frac{t}{t_c} \right) \]  

(13d)

The decomposition settlement during filling is given by

\[ U_d = \beta m'[1 - \frac{1}{k}(1 - e^{-kt})] \]  

(13e)

where \( U_d \) = decomposition settlement at time \( t \leq t_c \) after start of filling; \( m' \) = rate of increase of waste thickness with time (assumed uniform); \( k \) = decay constant during filling (assumed constant) = 0.003 to 0.21 as per U.S. EPA (1998); \( \beta \) = fraction of waste mass that can potentially be converted to gas; and \( k \) = decomposition factor. 

The postclosure decomposition settlement is given by

\[ (U_d)_{t=tc} = \beta H\left[ 1 - \frac{1}{tk'}(1 - e^{-kt})(1 - e^{-k't}) \right] \]  

(13f)

where \( (U_d)_{t=tc} \) = decomposition settlement at time \( t_{pc} \), larger or equal to \( t_c \) and \( k' \) = postfilling decay.

Liu et al. (2006) developed a model considering gas generation due to waste decomposition. The gas pressure within the landfill is elevated by the accumulation of the gas produced by waste decomposition. The flux of gas flow, which is induced by the increase of gas pressure, is calculated based on the concept of steady gas flow through an unsaturated medium. The settlement of landfill is assumed to be the sum of the decomposed solid proportion and the outflow proportion of gas. The amount of settlement is estimated by adopting a constitutive relationship of stress versus volume for an unsaturated soil. Gas pressure at different depths at an elapsed time \( t \) is given by

\[ U_d(\gamma,t) = \sum_{n=1}^{\infty} \left\{ \frac{4U_0}{(2n+1)\pi} e^{-\lambda_n\gamma} + \frac{4B}{(2n+1)(\lambda_n-k)\pi} \right\} \left( e^{k\gamma} - e^{-k\gamma} \right) \sin(\pi y) \]  

(14a)

where \( U_0 \) = excess pore-gas pressure (N·m⁻²); \( U_0 \) = pore-gas pressure at \( t=0 \) (N·m⁻²); \( n \) = porosity in the landfill; \( t \) = time; \( B \) = interactive constant associated with the biodegradation of organics; \( k \) = first-order degradation constant (3.3 × 10⁻⁵ to 3.8 × 10⁻³/day); and \( y \) = depth below the landfill surface. Other terms in the preceding equation are defined as follows:

\[ \lambda_n = A p^2 \]

\[ p = \frac{(2n+1)}{2L} \]

\[ A = \frac{V_0k_sRT}{\omega_0(V_0\beta_0 + V_a)g} \]

\[ B = \left( \frac{m-t}{m} \right) \frac{k_sR\beta_0}{\omega_0} \left( \frac{m}{V_0\beta_0 + V_a} + V_a \right) \]  

(14b)

where \( A \) = coefficient of compression with respect to the air phase; \( R \) = universal gas constant (N·m·mol⁻¹·K⁻¹); \( T \) = absolute temperature within landfill (K); \( w \) = water content of waste (25–60%); \( M_s \) = initial mass of biodegradable organics in waste (kg); \( V_0 \) = initial total volume of waste (m³); \( k_o \) = gas activity constant (0.01–0.15 m³/day); \( V_a \) = pore-gas volume in waste (m³); \( \omega_o \) = molecular mass of gas produced from biodegradation of organics (kg·mol⁻¹); \( m \) = coefficient of gas volume change; \( \omega_o \) = molecular mass of original air (kg·mol⁻¹); and \( e \) = strain of waste element. Other terms defined in this model are

\[ u_i = \frac{\rho_iRT}{\omega_i} \]

\[ u_b = \frac{\rho_bRT}{\omega_b} \]

\[ \rho_i = \frac{\mu_i\omega_i}{RT} \]

\[ \rho_b = \frac{M_i}{V_a}(1 - e^{-kh}) \]

\[ M_i = M_0 \cdot r_b \]  

(14c)

where \( u_i \) = initial gas pressure in the landfill element; \( u_b \) = gas pressure produced by biodegradation process; \( r_b \) = mass ratio of biodegradable organics (%); \( \rho_i \) = produced gas density (kg·m⁻³); \( \rho_i \) = original air density in waste (kg·m⁻³); and \( \rho_i \) = density of waste solid (kg·m⁻³).

**Proposed Constitutive Model for MSW**

Published models predict global settlement of MSW landfill; however, a mechanistic model that accounts for the stress-strain (constitutive) response of MSW is needed to predict stress-deformation-time response of MSW. The settlement behavior of MSW is influenced by the stiffness variations with time that are reflected in stress-strain response of MSW. The stiffness of MSW, defined in terms of appropriate moduli such as elastic modulus and shear modulus, is required to evaluate the settlement response.
with time. This is possible with the development of an appropriate constitutive model based on fundamental geomechanics principles. Recently, efforts have begun to develop constitutive models to describe stress-strain response of MSW (Machado et al. 2002, 2008). In this study, a simple constitutive model based on critical state soil mechanics framework extended to incorporate mechanical creep and biodegradation-induced compression is proposed. This model can be incorporated in numerical models to predict the variations of settlement under load with time based on the site-specific conditions.

The following assumptions are made in the development of the proposed constitutive model for MSW:

1. The mechanical behavior follows elastoplastic behavior following critical state soil model framework, with associated flow rule;
2. The secondary compression is governed by the time-dependent phenomenon in exponential function similar to Gibson and Lo’s (1961) is given by

\[ e_c = b \Delta p' (1 - e^{-c't'}) \]  

(15a)

where \( b \) = coefficient of mechanical creep; \( \Delta p' \) = change in mean effective stress; \( c \) = rate constant for mechanical creep; and \( t' \) = time since application of the stress increment;
3. The biological composition is related to time and the total amount of strain that can occur due to biological decomposition. The time-dependent biological degradation is proposed by Park and Lee (1997) and is given by

\[ e_b = E_{db} (1 - e^{-d't}) \]  

(15b)

where \( E_{db} \) = total amount of strain that can occur due to biological decomposition; \( d \) = rate constant for biological decomposition; and \( t' \) = time since placement of the waste in the landfill.

Let us consider the isotropic loading of MSW as shown in Fig. 1(a) on the \( e - \ln p' \) plot. If the material is normally consolidated at A, the isotropic loading will follow path AB. Let us now unload the sample to the mean effective pressure \( p'_A \). Because of the elastoplastic nature, the unloading path will not follow loading path AB. Instead, the material will follow path BD upon unloading. When the material is reloaded from pressure \( p'_A \) to \( p'_B \), it will usually follow the same path indicates elastic behavior. The slope of the loading path is denoted by \( \lambda \), and the slope of unloading-reloading path is denoted by \( \kappa \). The vertical distance AD shows the plastic component in the change in volume, and DE shows the elastic component of the change in volume. Now we can write the total change in void ratio (e) during the loading-unloading cycle as detailed in the following.

From Fig. 1(a), total change in void ratio during loading path AB

\[ e = e_A - e_B = \lambda \ln \left( \frac{p'_B}{p'_A} \right) = \lambda (\ln p'_B - \ln p'_A) \]  

(15c)

Change in void ratio in path BD

\[ e' = e_D - e_E = \kappa \ln \left( \frac{p'_B}{p'_A} \right) = \kappa (\ln p'_B - \ln p'_A) \]  

(15d)

Increment in total volumetric strain is given by

\[ d\varepsilon_v = -\frac{de}{1 + e_0} = \frac{\lambda}{1 + e_0} \frac{dp'}{p'} \]  

(15e)

The elastic volumetric strain \( d\varepsilon_v^e \) can be written as

\[ d\varepsilon_v^e = -\frac{de}{1 + e_0} = \frac{\lambda}{1 + e_0} \frac{dp'}{p'} \]  

(15f)

Hence, increment in plastic volumetric strain can be written as

\[ d\varepsilon_v^p = -\frac{de}{1 + e_0} = \frac{\lambda}{1 + e_0} \frac{dp'}{p'} \]  

(15g)

These formulations for increments in elastic and plastic volumetric strains are well established in critical state soil mechanics literature (Wood 1990). However, there is a need to extend the elastoplasticity concepts for MSW considering time-dependent mechanical creep and biological degradation.

**Extension of Existing Model**

In addition to elastic and plastic behavior of MSW, considering compression due to mechanical creep and biological decomposition, the total volumetric strain of the MSW is expressed as

\[ d\varepsilon_v = d\varepsilon_v^e + d\varepsilon_v^p + d\varepsilon_v^b \]  

(15h)

where \( d\varepsilon_v^e \), \( d\varepsilon_v^p \), and \( d\varepsilon_v^b \) = increments of volumetric strain due elastic, plastic, time-dependent mechanical creep, and biodegradation effects, respectively.

From Eq. (15a) increment in volumetric strain due to creep is written as

\[ d\varepsilon_v^c = cb \Delta p' e^{-c't'} dt' \]  

(15i)

From, Eq. (15b) increment in volumetric strain due biodegradation effect is written as

\[ d\varepsilon_v^b = \frac{de}{1 + e_0} = \frac{\lambda}{1 + e_0} \frac{dp'}{p'} \]  

(15j)

Fig. 1. (a) Consolidation behavior in \( e - \ln p' \) space; (b) yield locus in \( q-p' \) space
In the present case, \( t' \) (time since application of the stress increment) and \( t'' \) (time since placement of the waste in the landfill) are considered equal to \( t \). By substituting Eqs. (15f), (15g), (15l), and (15j) in Eq. (15h), the total strain increment is given by

\[
de_{v} = \frac{\kappa}{1 + e} p' + \left( \frac{\lambda - \kappa}{1 + e} \right) \frac{dp'}{p'} + 2 \eta d \eta \frac{dp'}{M^2 + \eta^2} + cb \Delta \sigma e^{-ct} dt + dE_{de} e^{-dt}
\]

(15k)

On simplification of Eq. (15k), total increment in volumetric strain becomes

\[
de_{v} = \frac{\lambda}{1 + e} p' + \left( \frac{\lambda - \kappa}{1 + e} \right) \frac{2 \eta d \eta}{M^2 + \eta^2} + cb \Delta \sigma e^{-ct} dt + dE_{de} e^{-dt}
\]

(15l)

Total change in volumetric strain in terms of void ratio written as

\[
q = Mp' \sqrt{\left[ \left( \frac{p'}{p'} \right)^{\lambda} \exp \left[ \frac{(e_0 - e)}{1 + e_0} + b \Delta \sigma e^{-ct} + dE_{de} e^{-dt} \right] (1 + e) \right]} \frac{1}{(1 - \kappa)}
\]

(16)

where \( p' \) = mean effective stress and \( p'_0 \) = preconsolidation pressure. A detailed derivation of Eq. (16) is given in Chouskey (2009) and Sivakumar Babu et al. (2010).

Eq. (16) is the proposed new model for MSW, which is an extended form of modified cam clay model that predicts the time-settlement behavior under loading. In addition to elastic and plastic strains, the total volumetric strain \( (\epsilon_v) \) includes mechanical compression under loading as well as mechanical creep and biological degradation effects represented by Eqs. (15a) and (15b). \( M \) is the frictional constant, \( (e_0) \) is the initial void ratio, and \( (e) \) is the void ratio after load increment. Thus Eq. (16) represents the deviatoric stress for MSW under load considering mechanical creep and biodegradation effects. Chouskey (2009) and Sivakumar Babu et al. (2010) used the experimental data of Reddy et al. (2009a,b) and showed that the proposed model captures the stress-strain response of MSW satisfactorily.

**Model Application**

To assess the applicability of the proposed constitutive model for MSW, a typical landfill is considered and the predicted settlements based on the proposed model are compared with the other published models.

**MSW Landfill Conditions**

A MSW landfill of 30-m height is selected, which is assumed to be filled in 10 layers each at a thickness of 3 m (Fig. 2). After completing the filling of the landfill, a final cover system is assumed to be constructed that consists of composite liner (compacted clay and geomembrane) overlain by a sand drainage layer and then a vegetative cover soil layer. The proposed and previously published models are used to predict the landfill settlement with time for 30 years after closure for planning the postclosure development at the site.

**Model Predictions**

The proposed constitutive model enables the calculation of strains (settlement) and yield surfaces that reflect the variation of shear strength (expressed in \( q-p' \) form). The model parameters are determined based on laboratory testing results reported by Reddy et al. (2009a,b) and published literature. For comparison purposes, the values of relevant parameters in all models are consid-

![Fig. 2. MSW landfill scenario for estimation of settlement versus time using different models](image-url)
Comparison with Other Models

Comparative study of different models helps in assessing reliability and limitations of a model for use in practice for specific site conditions. However, the accuracy of models is dictated by the landfill data, including values of model parameters representing the specific waste composition. Table 1 presents a summary of model parameters and the specific values of parameters used for this comparative study. Fig. 5 represents the time-settlement behavior, while Table 2 summarizes the magnitudes of total settlement obtained from different settlement models. The 15 models (including the proposed model) can be grouped as (1) soil mechanics-based models; (2) empirical models; (3) rheological models; and (4) models incorporating biodegradation.

The maximum settlement based on the soil mechanics-based model (or Sowers model) with the consideration of primary and secondary settlement is obtained as 6.54 m. Similarly, the Bjarnard and Eders model also does not consider the biodegradation settlement exclusively and the maximum settlement predicted is 15.34 m. Hossain and Gabr (2005) ignores mechanical compression, but takes into account biodegradation with its own coefficient similar to secondary compression and predicted time-dependent settlement of 3.65 m.

Among the empirical models, the hyperbolic model predicted the least value (0.53 m). Logarithmic function model predicted a value of 2.35 m and attenuation equation model predicted a low value of settlement of 1.33 m. Using power creep function the settlement predicted is 9.92 m. The Gibson and Lo model, which is a rheological model, gives the settlement as 14.84 m.

Biodegradation models incorporate waste degradation and in some cases gas generation is explicitly considered. The model of Hettiarachchi et al. (2009) considers the effect of immediate compression and biodegradation-induced settlement and predicts the highest settlement of 20.18 m. The Park and Lee (1997) model neglects primary compression due to applied stress. This model only considers the secondary compression and the compression due to biodegradation and the maximum settlement obtained as 2.02 m.

The composite rheological model and the composite compressibility model developed by Marques (2001) and Marques et al. (2003), respectively, estimate settlement in terms of the mechanical compression, secondary compression, and the biodegradation-induced compression. The total settlement calculated from these models were 10.45 m and 8.97 m, respectively. The Oweis (2006) model considers mechanical and decomposition processes and predicts settlement of 19.74 m.

The Liu et al. (2006) model that uses the unsaturated consolidation theory for prediction of long-term MSW landfill settlement predicts the long-term settlement in relation to the production of gas during biodegradation of MSW. This model only considers the effect of gas on the settlement and the settlement is predicted to be 6.31 m.

The settlement predicted from the proposed constitutive model, which accounts for immediate applied loading, mechanical creep, and time-dependent biodegradation is 9.09 m. The model requires parameters based on the consolidation-time response, shear strength characteristics, and biodegradation characteristics of MSW. The model is general and assumes that the effects of gas emissions and leachate generation are reflected directly in biodegradation parameters. This model also accounts for strength increase with time and the associated volumetric compression.

The predicted total settlements can vary significantly depending on the model used. Some models ignore mechanical compression as it occurs during or immediately after the waste placement and considered to be practically unimportant. However, time-dependent compression, due to mechanical creep and biodegradation, is critical as it affects the stability of the cover and drainage systems, appurtenant structures placed within the waste (e.g., gas extraction wells), and any structures constructed over the landfill.
**Table 1. Model Parameters and Their Values Used for Prediction of MSW Landfill Settlement**

<table>
<thead>
<tr>
<th>Settlement model</th>
<th>Parameters and values</th>
<th>Basis for the parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Soil mechanics-based model</strong>&lt;br&gt;(Sowers 1973)</td>
<td>$C_c = 0.106$</td>
<td>Hettiarachchi (2005)</td>
</tr>
<tr>
<td></td>
<td>$t_1 &lt; 2 \times 10^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t_2 = 2 \times 10^3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t_3 = 2 \times 10^4$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$C_0 = 0.035$</td>
<td>El-Fadel et al. (1999)</td>
</tr>
<tr>
<td></td>
<td>$t_4 = 1 - 25$ days</td>
<td></td>
</tr>
<tr>
<td><strong>Bjarngard and Edgers model</strong>&lt;br&gt;(Bjarngard and Edgers 1990)</td>
<td>$C_{ai} = 0.035$; $C_{aj} = 0.215$; $t_1 = 1$ to 25 days; $t_2 = 200$ days</td>
<td>Hettiarachchi (2005)</td>
</tr>
<tr>
<td></td>
<td>$C_c = 0.106$</td>
<td>El-Fadel et al. (1999)</td>
</tr>
<tr>
<td></td>
<td>$t_1 = 10 - 15$ days</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t_2 = 100 - 2,000$ days</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t_3 = 3,500$ days</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t_4 = $time for creep at the end of biological degradation</td>
<td></td>
</tr>
<tr>
<td><strong>Logarithmic function</strong>&lt;br&gt;(Yen and Scanlon 1975)</td>
<td>$\alpha = 0.0386$</td>
<td>Yen and Scanlon (1975)</td>
</tr>
<tr>
<td></td>
<td>$\beta = 0.0156$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t_c = 1$ month</td>
<td></td>
</tr>
<tr>
<td><strong>Power creep function</strong>&lt;br&gt;(Edil et al. 1990)</td>
<td>$M' = 1.7 \times 10^{-3}$ kPa</td>
<td>El-Fadel et al. (1999)</td>
</tr>
<tr>
<td></td>
<td>$N' = 0.50$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t_1 = 1$ day</td>
<td></td>
</tr>
<tr>
<td><strong>Hyperbolic function</strong>&lt;br&gt;(Ling et al. 1998)</td>
<td>$\rho_0 = 0.001$ m/day</td>
<td>Ling et al. (1998)</td>
</tr>
<tr>
<td></td>
<td>$S_{ah} = 1.140$ m</td>
<td></td>
</tr>
<tr>
<td><strong>Attenuation equation</strong>&lt;br&gt;(Coumoulos and Koryalos 1997)</td>
<td>$C_n = 0.02$ for $t = 1 - 20$ months</td>
<td>Coumoulos and Koryalos (1997)</td>
</tr>
<tr>
<td></td>
<td>$C_n = 0.07$ for $t = 40 - 60$ months</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$C_n = 0.25$ for $t &gt; 80$ months</td>
<td></td>
</tr>
<tr>
<td><strong>Gibson and Lo model</strong>&lt;br&gt;(Gibson and Lo 1961)</td>
<td>$a = 0.0001$</td>
<td>El-Fadel et al. (1999)</td>
</tr>
<tr>
<td></td>
<td>$b' = 0.00305$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\lambda / b' = 0.0009$</td>
<td></td>
</tr>
<tr>
<td><strong>Long-term biodegradation model</strong>&lt;br&gt;(Park and Lee 1997)</td>
<td>$C_n = 0.005$</td>
<td>Park and Lee (1997)</td>
</tr>
<tr>
<td></td>
<td>$k = 2.06 / \text{year} = 0.005643 / \text{day}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\varepsilon_{tot, dec} = 6.65%$</td>
<td></td>
</tr>
<tr>
<td><strong>Hettiarachchi et al. (2009)</strong></td>
<td>Time (days)</td>
<td>Hettiarachchi et al. (2009)</td>
</tr>
<tr>
<td></td>
<td>$&lt; 2 \times 10^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$2 \times 10^2 - 2 \times 10^3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$2 \times 10^3 - 2 \times 10^4$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$&gt; 2 \times 10^4$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Solids type</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{sj}$</td>
<td>$G_{si}$</td>
</tr>
<tr>
<td>Nondegradable</td>
<td>0.35</td>
<td>1.637</td>
</tr>
<tr>
<td>Slowly degradable</td>
<td>0.25</td>
<td>1.637</td>
</tr>
<tr>
<td>Moderately degradable</td>
<td>0.25</td>
<td>1.637</td>
</tr>
<tr>
<td>Highly degradable</td>
<td>0.15</td>
<td>1.637</td>
</tr>
<tr>
<td>Nondegradable</td>
<td>0.35</td>
<td>1.637</td>
</tr>
<tr>
<td><strong>Composite rheological model</strong>&lt;br&gt;(Marques 2001)</td>
<td>$C_c = 0.106$</td>
<td>Marques (2001)</td>
</tr>
<tr>
<td></td>
<td>$b = 5.72 \times 10^{-4}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$c = 1.79 \times 10^{-3}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$E_{0,h} = 0.1585$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$d = 1.14 \times 10^{-5}$</td>
<td></td>
</tr>
<tr>
<td><strong>Composite compressibility model</strong>&lt;br&gt;(Marques et al. 2003)</td>
<td>$H_j = 3.048$ m</td>
<td>Marques et al. (2003)</td>
</tr>
<tr>
<td></td>
<td>$C' = 0.106$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$E_{0,h} = 0.159$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$d = 1.14 \times 10^{-5}$</td>
<td></td>
</tr>
</tbody>
</table>
Simple models such as soil mechanics-based models or empirical models involve only a few number of model parameters. Often, simple compression tests are adequate to determine the values of these parameters. However, comprehensive models such as the proposed model require characterization of stress-strain behavior based on triaxial shear tests or other such tests.

Table 1. (Continued.)

<table>
<thead>
<tr>
<th>Settlement model</th>
<th>Parameters and values</th>
<th>Basis for the parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanical and decomposition model (Oweis 2006)</td>
<td>( H=30 \text{ m} )</td>
<td>Oweis (2006)</td>
</tr>
<tr>
<td></td>
<td>( C'_c=0.106; \ C'_s=0.02; )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( t_c=300 \text{ days;} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( k=0.05/\text{year}=0.004167/\text{day} )</td>
<td>(as per U.S. EPA 1998);</td>
</tr>
<tr>
<td></td>
<td>( \beta=0.2 ) (in absence of site-specific data)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( k'=\text{postfilling decay} \ (k'&lt;k \text{ if landfill is capped). Here } k'=k \text{ is assumed} )</td>
<td></td>
</tr>
<tr>
<td>Liu and Chen (Liu et al. 2006)</td>
<td>( U_0=1,500 \text{ N/m}^2 )</td>
<td>Liu et al. (2006)</td>
</tr>
<tr>
<td></td>
<td>( n=58.04% )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( k=3.0\times 10^{-4}/\text{day} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( k_a=0.04 \text{ m/day} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( w=30% )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \omega_0=0.030 \text{ kg/mol} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \omega_1=0.0288 \text{ kg/mol} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( T=310 \text{ K} )</td>
<td></td>
</tr>
<tr>
<td>Proposed model (this study)</td>
<td>( C_c=0.106 )</td>
<td>Sivakumar Babu et al. (2010)</td>
</tr>
<tr>
<td></td>
<td>( \lambda=0.046 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \kappa=0.0046 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( d'=24 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( G=0.97 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( e_0=0.528 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Water content=44%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( OCR=1.0 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( b=5.72\times 10^{-4} )</td>
<td>Chouskey (2009)</td>
</tr>
<tr>
<td></td>
<td>( c=1.79\times 10^{-3} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( E_{ds}=0.158 )</td>
<td>Reddy et al. (2009a,b)</td>
</tr>
<tr>
<td></td>
<td>( d=1.14\times 10^{-3} )</td>
<td></td>
</tr>
</tbody>
</table>

**Model Validation**

The accuracy of the model predictions can be better established by predicting the settlement of actual landfills where extensive settlement monitoring programs are being implemented. In absence of the field data, it is useful to compare the predictions from the proposed model with the predictions of reported models that are known to be validated based on the field measurements. Marques (2001) and Marques et al. (2003) applied their models to predict settlement and compared with the field measurements of MSW settlements in Brazil and showed that the latter model predicts the observed landfill settlements closely. Hence, to examine the applicability of the proposed model, predicted settlements from the model are compared with the predictions of models of Marques (2001) and Marques et al. (2003). Fig. 6 shows a comparison of settlement response obtained from the proposed model and that predicted by Marques (2001) and Marques et al. (2003) models. The predicted results from the proposed model indicate that the ultimate settlements are in the same range as those of predictions from Marques et al. (2003) model which considers lift thickness in the model. These observations lend credence to using the proposed model to predict settlement of landfills in the field. However, direct validation of the proposed model with the field measurements is needed to establish the accuracy of the model predictions.

Fig. 5. Comparison of long-term landfill settlement predictions based on different models.
Influence of Waste Lift Thickness

As pointed out by Marques et al. (2003), the waste lift thickness is an important aspect in landfilling operations and prediction of settlements. In order to examine the effect of lift thickness, the landfill profile selected for this study is assumed to be filled in different lift thicknesses varying from 0.50 to 3 m. The proposed model is used to predict the settlement with these different lift thicknesses and the results are shown in Fig. 7. It can be noted that the lift thickness has significant influence on the settlement response from the initial to ultimate settlement. For example, the initial settlement (at 1 day) varies from 3.02 to 5.8 m and the ultimate settlement value in 30 years (10,950 days) varies from 4.3 to 9.09 m as the lift thickness is increased from 0.50 to 3.0 m. These results show that the waste lift thickness during filling operations should be decreased in order to reduce the ultimate settlement.

Conclusions

Prediction of MSW landfill settlement is critical to design cover systems and ensure the safety of appurtenant structures placed within the waste (e.g., gas extraction wells/draws) and structures constructed over the landfill. MSW settles due to physical, chemical, and biological processes, and the total settlement is commonly assumed to consist of mechanical compression, me-

**Table 2. Comparative MSW Landfill Settlement Predictions Using Different Models**

<table>
<thead>
<tr>
<th>Settlement model</th>
<th>Primary settlement</th>
<th>Secondary settlement</th>
<th>Total settlement (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Soil mechanics-based models</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Soil mechanics-based model (Sowers 1973)</td>
<td>√</td>
<td>√</td>
<td>6.54</td>
</tr>
<tr>
<td>Bjarngard and Edgers model (Bjarngard and Edgers 1990)</td>
<td>√</td>
<td>√</td>
<td>15.34</td>
</tr>
<tr>
<td>Hossain and Gabr (2005)</td>
<td>√</td>
<td></td>
<td>3.65</td>
</tr>
<tr>
<td><strong>Empirical models</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Logarithmic function (Yen and Scanlon 1975)</td>
<td>√</td>
<td></td>
<td>2.35</td>
</tr>
<tr>
<td>Power creep function (Edil et al. 1990)</td>
<td>√</td>
<td></td>
<td>9.92</td>
</tr>
<tr>
<td>Hyperbolic function (Ling et al. 1998)</td>
<td>√</td>
<td></td>
<td>0.53</td>
</tr>
<tr>
<td>Attenuation equation (Coumoulos and Koryalos 1997)</td>
<td>√</td>
<td></td>
<td>1.33</td>
</tr>
<tr>
<td><strong>Rheological model</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gibson and Lo model (Gibson and Lo 1961)</td>
<td>√</td>
<td></td>
<td>14.84</td>
</tr>
<tr>
<td><strong>Settlement models incorporating biodegradation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long-term biodegradation model (Park and Lee 1997)</td>
<td>√</td>
<td></td>
<td>0.242</td>
</tr>
<tr>
<td>Hettiarachchi et al. (2009)</td>
<td>√</td>
<td></td>
<td>20.18</td>
</tr>
<tr>
<td>Composite rheological model (Marques 2001)</td>
<td>√</td>
<td></td>
<td>10.45</td>
</tr>
<tr>
<td>Composite compressibility model (Marques et al. 2003)</td>
<td>√</td>
<td></td>
<td>8.97</td>
</tr>
<tr>
<td>Mechanical and decomposition (Oweis 2006)</td>
<td>√</td>
<td></td>
<td>19.74</td>
</tr>
<tr>
<td>Liu et al. (2006)</td>
<td>√</td>
<td></td>
<td>6.31</td>
</tr>
<tr>
<td>Proposed model (this study)</td>
<td>√</td>
<td></td>
<td>9.09</td>
</tr>
</tbody>
</table>

**Fig. 6.** Comparison of long-term landfill settlement based on the proposed model and Marques (2001) and Marques et al. (2003) models

**Fig. 7.** Effect of waste lift thickness on predicted settlement using the proposed model
mechanical creep, and biodegradation-induced compression. Several models have been developed based on different assumptions and they are grouped under soil mechanics based, empirical, or biodegradation based models. In this study, a constitutive model which incorporates mechanical compression, mechanical creep, and biodegradation-induced compression is developed based on critical state soil mechanics principles and first-order degradation rate. This model can be used in advanced numerical methods to predict the settlement behavior of MSW landfills. In order to compare the proposed model with the reported simple models, settlement is predicted for a typical landfill for a total time period of 30 years. The results show that the predicted settlement can vary significantly depending on the model selected and the specific values of model parameters used. The proposed constitutive model predicts the total settlement in a range similar to models that consider all three components of the settlement. The reduced waste lift thickness is shown to reduce the ultimate settlement. Further validation of the models is required based on field measured settlement at different landfill sites.

Acknowledgments

This project is a collaborative effort between the Indian Institute of Science, Bangalore, India and the University of Illinois at Chicago. Partial funding is provided by the U.S. National Science Foundation (Grant No. CMMI #0600441), which is gratefully acknowledged. The writers thank the reviewers for critical and useful comments.

Notation

The following symbols are used in this paper:

- \( A \) = coefficient of compression with respect to the air phase;
- \( a \) = primary compressibility parameter;
- \( B \) = interactive constant associate with the biodegradation of organics;
- \( b \) = coefficient of secondary mechanical compression;
- \( b' \) = secondary compressibility parameter;
- \( C_{a1} \) = secondary compression index;
- \( C_{a2} \) = secondary compression ratio;
- \( C_{a3} \) = creep index;
- \( C_{a4} \) = compression index which is a function of stress level and degree of decomposition;
- \( C_{a5} \) = intermediate secondary compression ratio;
- \( C_{a6} \) = long-term/final secondary compression ratio, respectively;
- \( C_{b1} \) = biodegradation index;
- \( C_{b2} \) = coefficient of secondary compression;
- \( C_{b3} \) = compressibility parameter;
- \( C_{b4} \) = primary compression ratio;
- \( c \) = secondary mechanical compression rate;
- \( d \) = secondary biological compression rate;
- \( e_i \) = increment in volumetric strain due to biodegradation;
- \( e_i' \) = increment in volumetric strain due to creep;
- \( e_i'' \) = increment in plastic shear strain;
- \( e_i''' \) = increment in plastic volumetric strain;
- \( E_{do} \) = total compression due waste degradation;
- \( e_0 \) = initial void ratio;
- \( f_{ij} \) = initial solids fraction for each waste;
- \( G_{ij} \) = specific gravity of jth group of waste solids;
- \( H \) = initial thickness of waste layer;
- \( H_0 \) = initial height of waste;
- \( H_t \) = initial height of the landfill;
- \( H_i \) = initial thickness of compacted lift \( i \);
- \( k \) = decay constant during filling;
- \( k_a \) = initial total volume of waste;
- \( k_1 \) = first-order decomposition strain rate;
- \( k' \) = postfilling decay;
- \( k'' \) = gas activity constant;
- \( M \) = frictional constant;
- \( M_{d} \) = design total mass accepted at landfill;
- \( M' \) = reference compressibility;
- \( m_3 \) = coefficient of gas volume change;
- \( m' \) = rate of filling;
- \( N \) = number of lifts in the landfill;
- \( N' \) = rate of compression;
- \( p' \) = mean effective stress;
- \( p_0 \) = preconsolidation pressure;
- \( R \) = universal gas constant;
- \( r_b \) = porosity in the landfill;
- \( r_b \) = mass ratio of biodegradable organics;
- \( T \) = absolute temperature within landfill \( K \);
- \( t \) = time since beginning of filling;
- \( t_c \) = construction period;
- \( t_e \) = time of completion of filling;
- \( t_{es} \) = time usually assumed as 1 month;
- \( t_i \) and \( t_j \) = times at which lifts \( i \) and \( j \), respectively;
- \( t_l \) = time since load application;
- \( t_f \) = reference time;
- \( t_i \) = time for completion of initial compression;
- \( t_2 \) = ending time period for which long-term settlement of layer \( i \) is;
- \( t_3 \) = time for total period of time considered in modeling;
- \( t_4 \) = time for the creep at the end of biological degradation;
- \( t' \) = time elapsed since loading application;
- \( t_2 \) = time for intermediate secondary compression;
- \( t_3 \) = time for completion for biological compression;
- \( t' \) = time elapsed since waste disposal;
- \( U_a \) = excess pore-gas pressure;
- \( U_d \) = decomposition settlement;
- \( (U_d)_{ir} \) = decomposition settlement;
- \( U_p \) = primary settlement of waste for design thickness;
- \( U_0 \) = pore-gas pressure;
- \( u_b \) = gas pressure produced by biodegradation process;
- \( u_i \) = initial pore-gas pressure;
- \( V_a \) = pore-gas volume in waste;
- \( w \) = water content;
- \( Y \) = vertical strain rate;
- \( y \) = depth below the landfill surface;
- \( \alpha \) = fitting parameter;
- \( \beta \) = fitting parameter;
- \( \beta' \) = fraction of waste mass that can potentially be converted to gas;
- \( \gamma_i \) = unit weight of lift \( i \);
- \( \Delta H \) = settlement due to primary and secondary consolidation;
- \( \Delta \sigma \) = increment of overburden pressure acting at the midlevel of the;
\[ \Delta \sigma' = \text{difference in effective stress;} \]
\[ \delta \varepsilon_\nu = \text{increment in volumetric strain;} \]
\[ \delta \varepsilon_\sigma = \text{increment in shear strain;} \]
\[ \varepsilon = \text{strain of waste element;} \]
\[ \varepsilon_{ti} = \text{time-dependent strain due to biological decomposition;} \]
\[ \varepsilon_{bi} = \text{strain at time } t \text{ in lift } i \text{ due to biological decomposition;} \]
\[ \varepsilon_c = \text{time-dependent strain due to mechanical creep;} \]
\[ \varepsilon_{ct} = \text{strain at time } t \text{ in lift } i \text{ due to mechanical creep;} \]
\[ \varepsilon_{p} = \text{strain due to instantaneous response to applied load;} \]
\[ \varepsilon_{pl} = \text{strain in lift } i \text{ resulting from instantaneous response to loading;} \]
\[ \varepsilon_{tot, dec} = \text{total amount of compression that will occur due to decomposition;} \]
\[ \eta = \text{stress ratio;} \]
\[ \kappa = \text{recompression index;} \]
\[ \lambda = \text{compression ratio;} \]
\[ \lambda_j = \text{first-order kinetic constant for the } j\text{th group;} \]
\[ \lambda/b' = \text{rate of secondary compression;} \]
\[ \rho_0 = \text{produced as density;} \]
\[ \rho_i = \text{original air density in waste;} \]
\[ \rho_s = \text{density of waste solid;} \]
\[ \rho_w = \text{density of water;} \]
\[ \rho_0 = \text{initial rate of settlement;} \]
\[ \sigma_N = \text{vertical stress;} \]
\[ \sigma_i = \text{existing overburden pressure acting at mid level of the layer;} \]
\[ \sigma' = \text{effective stress;} \]
\[ \phi' = \text{friction angle;} \]
\[ \omega_0 = \text{molecular mass of produced gas from biodegradation of organics; and} \]
\[ \omega = \text{molecular mass of original air.} \]

References


